Microwave circuit characterization: Back to basics !!

Notations and conventions

- Harmonic signals => Maxwell in frequency domain

- Phasor definition

Maxwell

$$\nabla \times \mathbf{E}(t,\mathbf{r}) = -\frac{\partial \mathbf{B}(t,\mathbf{r})}{\partial t}$$

$$\nabla \times \mathbf{H}(t,\mathbf{r}) = \frac{\partial \mathbf{D}(t,\mathbf{r})}{\partial t} + \mathbf{J}(t,\mathbf{r})$$

$$\nabla \cdot \mathbf{D}(t,\mathbf{r}) = \rho(t,\mathbf{r})$$

$$\nabla \cdot \mathbf{B}(t,\mathbf{r}) = 0$$

Time domain

$$\frac{\partial \rho(t,\mathbf{r})}{\partial t} + \nabla \cdot \mathbf{J}(t,\mathbf{r}) = 0$$

$$\nabla \times \underline{\mathbf{E}}(\mathbf{r}) = -j\omega \underline{\mathbf{B}}(\mathbf{r})$$

$$\nabla \times \underline{\mathbf{H}}(\mathbf{r}) = j\omega \underline{\mathbf{D}}(\mathbf{r}) + \underline{\mathbf{J}}(\mathbf{r})$$

$$\nabla \cdot \underline{\mathbf{D}}(\mathbf{r}) = \underline{\rho}(\mathbf{r})$$

$$\nabla \cdot \underline{\mathbf{B}}(\mathbf{r}) = 0$$

Frequency domain

$$\nabla \cdot \underline{\mathbf{J}}(\mathbf{r}) + j\omega \underline{\rho}(\mathbf{r}) = 0$$

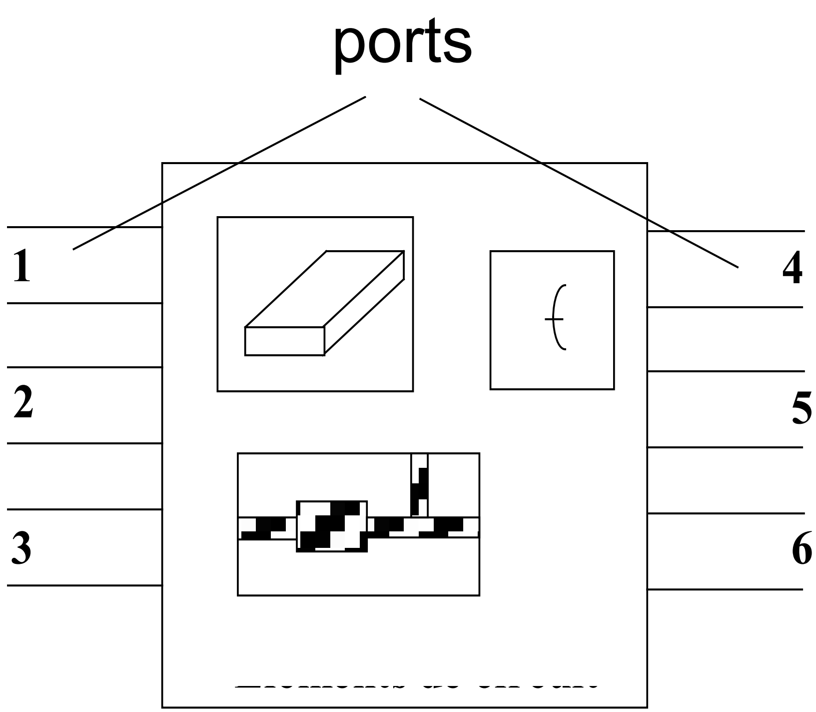
Phasor definition

$$A\cos(\omega t+\phi)=\mathrm{Re}\,[Ae^{j\phi}\exp(j\omega t)]=\sqrt{2}\,\mathrm{Re}\,[\frac{A}{\sqrt{2}}e^{j\phi}\exp(j\omega t)]=\sqrt{2}\,\mathrm{Re}\,[A_e e^{j\phi}\exp(j\omega t)]$$

physical values f(**r**,t) (f**=E, D, H, B, J,***r*) are replaced by <u>f(**r**)</u> (f**=**<u>**E**</u>, <u>**D**</u>, <u>**H**</u>, <u>**B**</u>, <u>**J**</u>, <u>r</u>), using the relation

$$f(t,r)=\sqrt{2}\,\mathrm{Re}\left[\underline{f(r)}e^{j\omega t}\right]$$

Microwave component

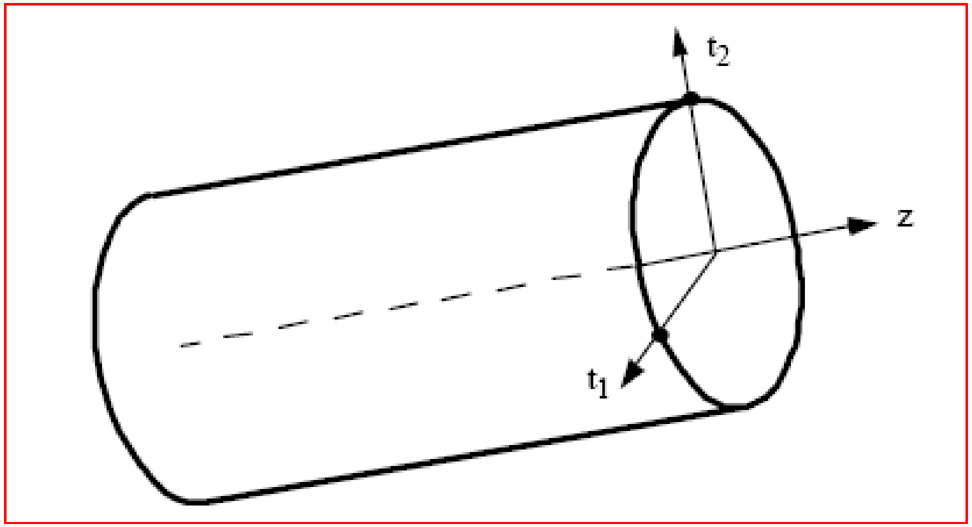


Circuit elements

Key point: modes

- We want to solve Maxwell's equation along a general transmission line
- The structure is invariant along the direction of propagation
- Thus the wave equation (Helmoltz equation) can be separated in a transverse and longitudinal part

Guided propagation



for all vector **v**,
we can write :
$v = v_z \mathbf{z} + \mathbf{v_t}$

Structure is invariant in z

v_z is the longitudinal component, $\mathbf{v_t}$ the transverse component

The vector operators become :

$$\nabla = \frac{\partial}{\partial z}\hat{\mathbf{z}} + \nabla_{\mathbf{t}}$$

$$\nabla^2 = \frac{\partial^2}{\partial z^2} + \nabla_t^2$$

Guided propagation

All function f(t1,t2,z) will be considered as the product (separation of variables) :

$$f(t_1, t_2, z) = T(t_1, t_2) Z(z)$$

The wave equation becomes :

$$\left(\nabla^2 + k^2\right) f = 0 \quad => \quad \frac{\nabla_t^2 T}{T} + k^2 + \frac{1}{Z}\frac{d^2 Z}{dz^2} = 0$$

Guided propagation

This relation has to be valid for all (t_1, t_2, z), thus

$$\frac{\nabla_t^2 T}{T} + k^2 = -\gamma^2$$

$$\frac{1}{Z}\frac{d^2 Z}{dz^2} = -\gamma^2$$

The longitudinal dependency has an easy analytical solution :

$$Z(z) = Ae^{-\gamma z} + Be^{+\gamma z}$$

It is the superposition of an incident wave and a reflected wave

Guided propagation

γ : longitudinal propagation constant

$$\gamma = \alpha + j\beta$$

α : lineic attenuation [neper/m] or [dB/m]
β : lineic phase shift [rad/m]

Guided propagation

If we consider only incident waves, B=0.
We can then write for the electromagnetic fields :

$$\mathbf{E}(t_1,t_2,z)=\mathbf{e}(t_1,t_2)e^{-\gamma z}$$

$$\mathbf{H}(t_1,t_2,z)=\mathbf{h}(t_1,t_2)e^{-\gamma z}$$

The transverse part will of course depend on the transverse geometry of the guide

Transverse dependency

$$\left(\nabla_t^2 + k^2 + \gamma^2\right)\mathbf{e} = \left(\nabla_t^2 + k_c^2\right)\mathbf{e}$$

$$\left(\nabla_t^2 + k^2 + \gamma^2\right)\mathbf{h} = \left(\nabla_t^2 + k_c^2\right)\mathbf{h}$$

$$k_c^2 = k^2 + \gamma^2$$

The admissible k_c are the eigenvalues of the guide, the corresponding **e** and **h** the eigenmodes of the guide

longitudinal propagation constant

A propagation constant corresponds to each eigenvalue :

$$\gamma = \sqrt{k_c^2 - k^2}$$

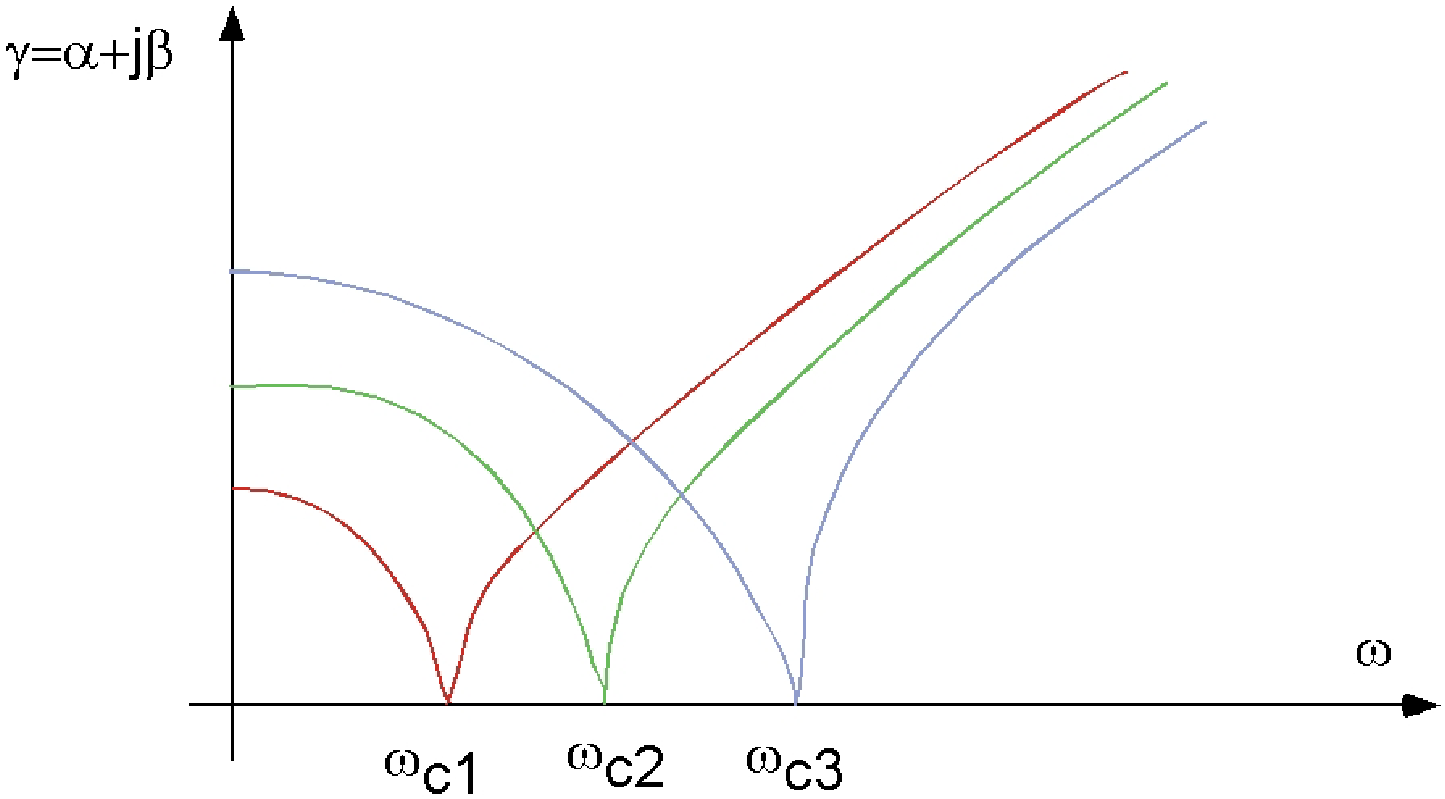
Thus, in a lossless case, depending on the relation between k and k_c the longitudinal propagation constant can be real or imaginary :

$$\text{if } k_c > \omega\sqrt{\varepsilon\mu} \quad \text{then } \gamma = \alpha$$

$$\text{if } k_c = \omega\sqrt{\varepsilon\mu} \quad \text{then } \gamma = 0$$

$$\text{if } k_c < \omega\sqrt{\varepsilon\mu} \quad \text{then } \gamma = j\beta$$

Dispersion diagram



Guided propagation

The electromagnetic fields have, in general, three components. The modes also :

$$\mathbf{E}(t_1,t_2,z)=E_{t_1}(t_1,t_2,z)\hat{\mathbf{t}}_\mathbf{1}+E_{t_2}(t_1,t_2,z)\hat{\mathbf{t}}_\mathbf{2}+E_z(t_1,t_2,z)\hat{\mathbf{z}}$$

$$\mathbf{H}(t_1,t_2,z)=H_{t_1}(t_1,t_2,z)\hat{\mathbf{t}}_\mathbf{1}+H_{t_2}(t_1,t_2,z)\hat{\mathbf{t}}_\mathbf{2}+H_z(t_1,t_2,z)\hat{\mathbf{z}}$$

$$\mathbf{e}(t_1,t_2)=e_{t_1}(t_1,t_2)\hat{\mathbf{t}}_\mathbf{1}+e_{t_2}(t_1,t_2)\hat{\mathbf{t}}_\mathbf{2}+e_z(t_1,t_2)\hat{\mathbf{z}}$$

$$\mathbf{h}(t_1,t_2)=h_{t_1}(t_1,t_2)\hat{\mathbf{t}}_\mathbf{1}+h_{t_2}(t_1,t_2)\hat{\mathbf{t}}_\mathbf{2}+h_z(t_1,t_2)\hat{\mathbf{z}}$$

And we can write

$$\mathbf{E}(t_1,t_2,z)=(e_z\hat{\mathbf{z}}+\mathbf{e_t})e^{-\gamma z}$$

$$\mathbf{H}(t_1,t_2,z)=(h_z\hat{\mathbf{z}}+\mathbf{h_t})e^{-\gamma z}$$

Guided propagation

The 6 componants of the fields are not independent, the six components of the modes neither. Indeed, through Maxwell's equations we can write:

$$\nabla_{\mathbf{t}} \times \mathbf{e}_{\mathbf{t}} = -j\omega\mu\left(h_z \hat{\mathbf{z}}\right)$$

$$\nabla_{\mathbf{t}} \times \mathbf{h}_{\mathbf{t}} = j\omega\varepsilon\left(e_z \hat{\mathbf{z}}\right)$$

and we get finally (after some manipulations) :

$$k_c^2 \mathbf{e}_{\mathbf{t}} = -\gamma \nabla_t e_z + j\omega\mu \hat{\mathbf{z}} \times \nabla_t h_z$$

$$k_c^2 \mathbf{h}_{\mathbf{t}} = -\gamma \nabla_t h_z - j\omega\varepsilon \hat{\mathbf{z}} \times \nabla_t e_z$$

Guided propagation

Thus, for a given problem, we need only to compute e_z and h_z.
The other components will be obtained using Maxwell's equations

Strategy

1. Compute e_z and h_z solving :
2. Compute the longitudinal propagation constant. For an incident wave, select the root with $\mathrm{Im}(\gamma)>0$
3. Compute the transverse components of the modes
4. Reconstruct the electromagnetic fields
5. Do the same thing for the reflected wave, replacing γ by $-\gamma$

$$\left(\nabla_t^2 + k_c^2\right)\mathrm{e}_z = 0 \;;\; \left(\nabla_t^2 + k_c^2\right)h_z = 0$$

$$\gamma = \alpha + j\beta = \sqrt{k_c^2 - k^2}$$

$$k_c^2\mathbf{e_t} = -\gamma\nabla_t e_z + j\omega\mu\hat{\mathbf{z}}\times\nabla_t h_z$$

$$k_c^2\mathbf{h_t} = -\gamma\nabla_t h_z - j\omega\varepsilon\hat{\mathbf{z}}\times\nabla_t e_z$$

$$\mathbf{E}(t_1,t_2,z) = (e_z\hat{\mathbf{z}} + \mathbf{e_t})e^{-\gamma z}$$

$$\mathbf{H}(t_1,t_2,z) = (h_z\hat{\mathbf{z}} + \mathbf{h_t})e^{-\gamma z}$$

Helmholz' equation

$$\left(\nabla_t^2 + k_c^2\right)\mathrm{e}_z = 0 \;;\; \left(\nabla_t^2 + k_c^2\right)h_z = 0$$

	TEM Modes	TM or E Modes	TE or H Modes	Hybrid Modes
ez	0	not 0	0	not 0
hz	0	0	not 0	not 0

Examples

- coaxial cable : TEM, TE, TM
- optical fibres : Hybrid modes
- waveguides : TE, TM
- Microstrip line: quasi TEM mode, hybrid modes
- Stripline: TEM TE and TM modes
- Coplanar waveguide: quasi TEM mose, hybrid modes

TEM mode

- characteristic equation :

$$e_z = h_z = 0$$

- second stage :

$$k_c^2 \mathbf{e_t} = -\gamma \nabla_t e_z + j\omega\mu \hat{\mathbf{z}} \times \nabla_t h_z = 0$$

$$k_c^2 \mathbf{h_t} = -\gamma \nabla_t h_z - j\omega\varepsilon \hat{\mathbf{z}} \times \nabla_t e_z = 0$$

TEM Mode

- Two solutions :

$$\mathbf{e_t} = \mathbf{h_t} = 0 \ \ \text{(trivial)}$$

$$k_c = 0 \quad => \ \gamma = jk = j\omega\sqrt{\varepsilon\mu}$$

TEM Mode

- We have to solve Maxwell's equations :

$$\nabla_t \times \mathbf{e_t} = 0 \qquad -\gamma\hat{\mathbf{z}} \times \mathbf{e_t} = -j\omega\mu\mathbf{h_t}$$

$$\nabla_t \times \mathbf{h_t} = 0 \qquad -\gamma\hat{\mathbf{z}} \times \mathbf{h_t} = j\omega\varepsilon\mathbf{e_t}$$

- Laplace's equation :

$$\mathbf{e_t} = -\nabla_t V \quad with \quad \nabla_t^2 V = 0$$

TEM Mode

- The magnetic field is given by :

$$\mathbf{h_t} = \frac{\gamma}{j\omega\mu}\hat{\mathbf{z}} \times \mathbf{e_t} = \frac{j\omega\varepsilon}{\gamma}\hat{\mathbf{z}} \times \mathbf{e_t}$$

- The wave impedance is given by :

$$Z_{\text{mod}} = \frac{j\omega\mu}{\gamma} = \frac{\gamma}{j\omega\varepsilon} = \sqrt{\frac{\mu}{\varepsilon}}$$

TEM Mode

- univoque definition of the current and voltage
- Fields identical to static fields
- zero cutoff frequency
- Needs at least two isolated conductors

TM modes

- The equation to be solved is :

$$\left(\nabla_t^2 + k_c^2\right)\mathrm{e}_z = 0$$

- The other components are obtained by :

$$k_c^2 \mathbf{e_t} = -\gamma \nabla_t e_z \quad \mathbf{h_t} = \frac{j\omega\varepsilon}{\gamma} \hat{\mathbf{z}} \times \mathbf{e_t}$$

TM modes

- Longitudinal propagation factor :

$$\gamma = \sqrt{k_c^2 - \omega^2 \varepsilon\mu}$$

- Wave impedance :

$$Z_{\text{mod}} = \frac{|\mathbf{e_t}|}{|\mathbf{h_t}|} = \frac{\gamma}{j\omega\varepsilon} = \frac{\sqrt{\omega^2 \mu\varepsilon - k_c^2}}{\omega\varepsilon}$$

TE modes

- The equation to be solved is :

$$\left(\nabla_t^2 + k_c^2\right) h_z = 0$$

- The other components are obtained by :

$$k_c^2 \mathbf{h_t} = -\gamma \nabla_t h_z \quad \mathbf{e_t} = \frac{j\omega\mu}{\gamma} \mathbf{h_t} \times \hat{\mathbf{z}}$$

TE modes

- Longitudinal propagation factor :

$$\gamma = \sqrt{k_c^2 - \omega^2 \varepsilon\mu}$$

- Wave impedance :

$$Z_{\text{mod}} = \frac{|\mathbf{e_t}|}{|\mathbf{h_t}|} = \frac{j\omega\mu}{\gamma} = \frac{\omega\mu}{\sqrt{\omega^2\mu\varepsilon - k_c^2}}$$

Longitudinal propagation coefficient

$$\gamma = \alpha + j\beta = k_c\sqrt{1-\left(\frac{\omega}{\omega_c}\right)^2} = \frac{\omega_c}{c}\sqrt{1-\left(\frac{\omega}{\omega_c}\right)^2}$$

Group velocity

$$v_g = \left(\frac{\partial \beta}{\partial \omega} \right)^{-1} = c\sqrt{1 - \left(\frac{\omega_c}{\omega} \right)^2}$$

phase velocity

$$v_\varphi = \frac{\omega}{\beta} = \frac{c}{\sqrt{1-\left(\dfrac{\omega_c}{\omega}\right)^2}}$$

guided wavelength

$$\lambda_g = \frac{2\pi}{\beta} = \frac{2\pi}{\sqrt{k^2 - k_c^2}} = \frac{\lambda}{\sqrt{1 - \left(\frac{\lambda}{\lambda_c}\right)^2}}$$

with

$$\lambda_c = \frac{2\pi}{k_c}$$

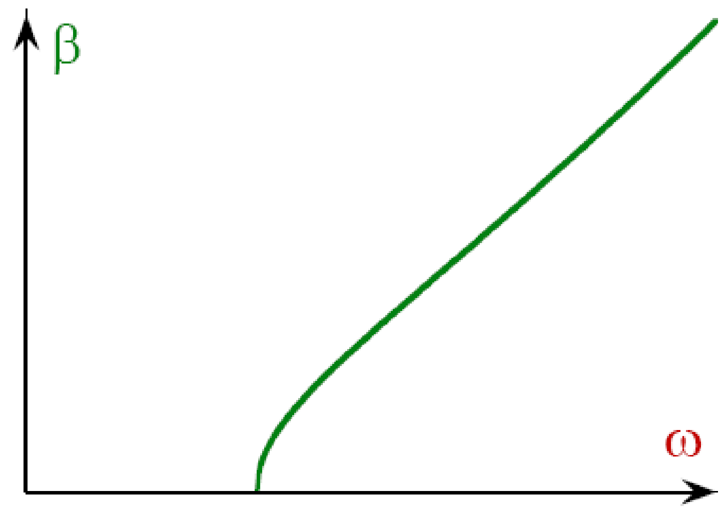
	TEM Modes	TM Modes	TE Modes
Basic equation	$\nabla_t^2 V = 0$	$\left(\nabla_t^2 + k_c^2\right) e_z = 0$	$\left(\nabla_t^2 + k_c^2\right) h_z = 0$
γ	$\sqrt{-\omega^2 \varepsilon\mu} = j\omega\sqrt{\varepsilon\mu}$	$\sqrt{k_c^2 - \omega^2 \varepsilon\mu}$	$\sqrt{k_c^2 - \omega^2 \varepsilon\mu}$
Ez	0	$e_z e^{-\gamma z}$	0
Hz	0	0	$h_z e^{-\gamma z}$
Et	$-\nabla_t V e^{-\gamma z}$	$-\left(\frac{\gamma}{k_c^2}\right)\nabla_t E_z$	$\left(\frac{j\omega\mu}{\gamma}\right)(\mathbf{H_t} \times \hat{\mathbf{z}})$
Ht	$\left(\frac{\gamma}{j\omega\mu}\right)(\hat{\mathbf{z}} \times \mathbf{E_t})$	$\left(\frac{j\omega\varepsilon}{\gamma}\right)(\hat{\mathbf{z}} \times \mathbf{E_t})$	$-\left(\frac{\gamma}{k_c^2}\right)\nabla_t H_z$
Zmod = |**et**| / |**ht**|	$\sqrt{\frac{\mu}{\varepsilon}}$	$\frac{\sqrt{\omega^2 \varepsilon\mu - k_c^2}}{\omega\varepsilon}$	$\frac{\omega\mu}{\sqrt{\omega^2 \varepsilon\mu - k_c^2}}$

Dispersion and distorsion (1)

When the linear phase shift β is a non-linear function of the frequency, the propagation is sais to be dispersive.

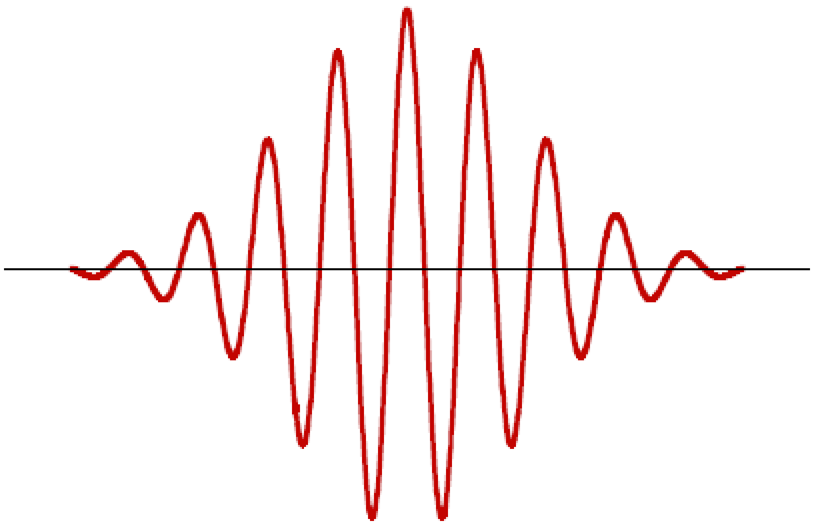
Waveguides and optical fibres are dispersive propagation lines

When a signal propagates on a dispersive line, it sustains a distorsion.



This effect is illustrated here with a "gaussian pulse" — where the developpments is mathematically simple

Dispersion and distorsion (2)



Modulated Gaussian pulse

$$f(t,z=0)=\cos(\omega_0 t)\,e^{-\frac{1}{2}\left(t/\tau\right)^2}$$

where 2τ is the width of the pulse at a level $1/\sqrt{e}=0,606$ of the maximum.

The Fourier transform of the gaussian pulse (spectrum of the pulse in the frequency domain) has also a gaussian dependency

$$\underline{F}(\omega,z=0)=\tau\sqrt{2\pi}\,e^{-\frac{1}{2}\left[\tau(\omega-\omega_0)\right]^2}$$

Dispersion and distorsion (3)

The signal propagates along the dispersive (dispersive) line and sustains a phase shift βz

$$\underline{F}(\omega, z) = \tau\sqrt{2\pi}\, \mathrm{e}^{-\frac{1}{2}[\tau(\omega-\omega_0)]^2}\, \mathrm{e}^{-\mathrm{j}\beta z}$$

The corresponding function in time domain is found by taking the inverse Fourier transform

$$f(t,z) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \underline{F}(\omega, z)\mathrm{e}^{\mathrm{j}\omega t}\mathrm{d}\omega = \frac{\tau\sqrt{2\pi}}{2\pi}\int_{-\infty}^{+\infty} \mathrm{e}^{-\frac{1}{2}[\tau(\omega-\omega_0)]^2}\, \mathrm{e}^{-\mathrm{j}\beta z}\mathrm{e}^{\mathrm{j}\omega t}\mathrm{d}\omega$$

But β is not a simplefunction of ω, thus this integral cannot be evaluated in a "simple" way ...

Dispersion and distorsion (4)

The spectrum of the pulse is usually narrow, thus we take a Taylor series developpment of $\beta(\omega)$ around ω_0

$$\beta = \beta_0 + \beta_1(\omega - \omega_0) + \frac{\beta_2}{2}(\omega - \omega_0)^2 + \dots \quad \text{avec} \quad \beta_0 = \beta(\omega_0) \text{ et } \beta_n = \left.\frac{\partial^n \beta}{\partial \omega^n}\right|_{\omega=\omega_0}$$

The term under the integral takes the form

$$e^{-\frac{1}{2}[\tau(\omega-\omega_0)]^2} e^{-j\left[\beta_0+\beta_1(\omega-\omega_0)+\frac{\beta_2}{2}(\omega-\omega_0)^2\right]z} e^{j\omega t}$$

regrouping the terms in ω, we obtain a term $e^{j\omega t}e^{-j\beta_1\omega z} = e^{j\omega t'}$, with $t' = t - \beta_1 z$, which corresponds to a translation with a velocity $1/\beta_1 = v_g$ (group velocity).

Dispersion and distorsion (5)

Grouping the terms in $(\omega-\omega_0)^2$, we get

$$e^{-\frac{1}{2}\left[\tau(\omega-\omega_0)\right]^2-j\left[\frac{\beta_2}{2}(\omega-\omega_0)^2\right]z}=e^{-\frac{(\omega-\omega_0)^2}{2}\left[\tau^2+j\beta_2 z\right]}=e^{-\frac{(\omega-\omega_0)^2\tau_e^2}{2}}$$

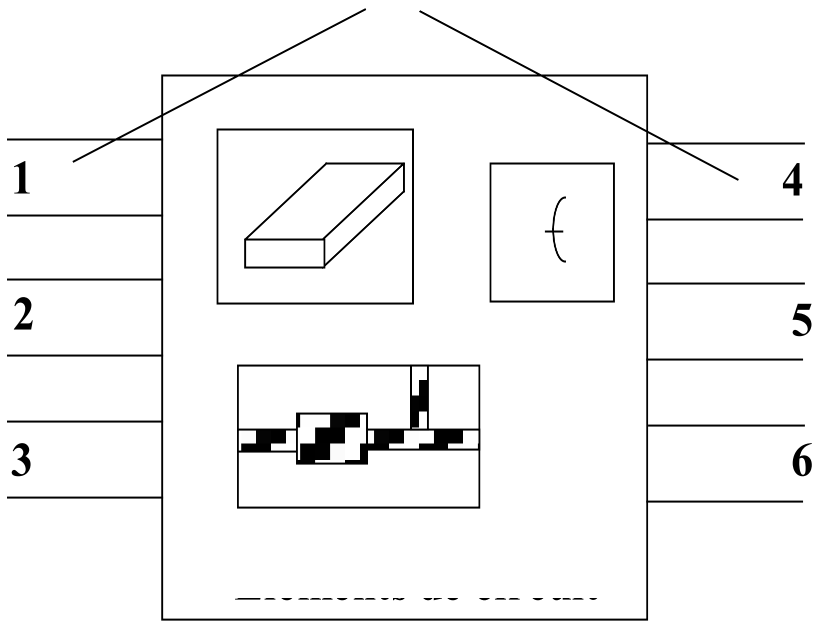
Taking the inverse Fourier transfom by making some approximations, we find that the width of the pulse becomes

$$\tau' \cong \frac{\left|\tau_e^2\right|}{\tau}=\sqrt{\tau^2+\left(\frac{\beta_2 z}{\tau}\right)^2}$$

At a large distance z, if the dispersion term β_2 is important, we tend towards $\tau' \cong \beta_2 z/\tau$: a very narrow pulse in $z = 0$ broadens faster then a wider pulse in $z = 0$.

Microwave component

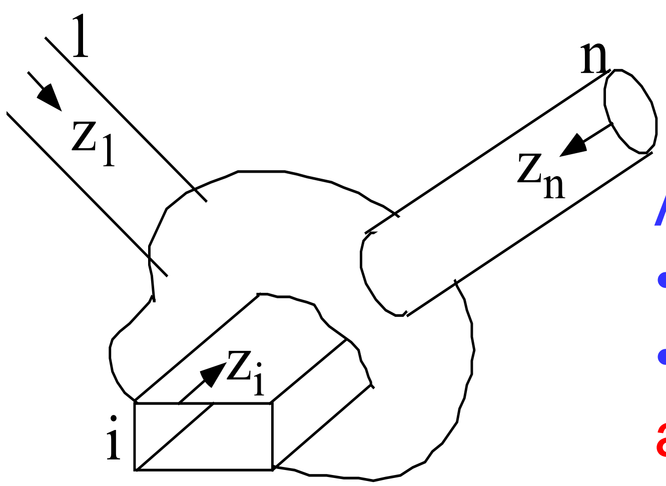
ports



Circuit elements

Reference planes

On each access line i of a component, a coordinate axis z_i is defined. The origin of this axis lies in the reference plane of the port i.



Assumptions :

- The transmission lines are lossless
- They support only the dominant mode or any other single mode
- The reference plane is distant enough from discontinuities to ensure that the None relevant modes are attenuated

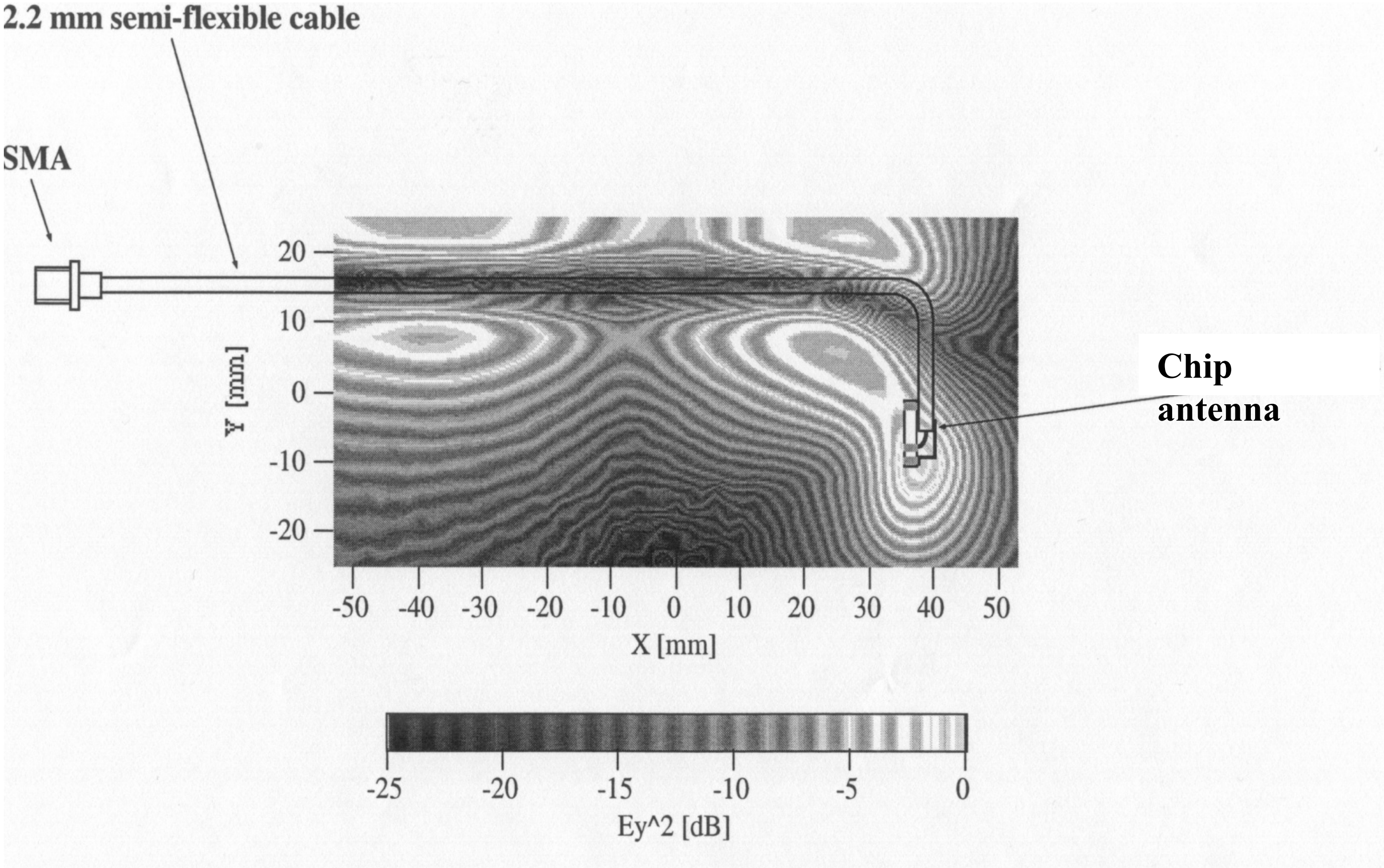
Point of interest : ill defined ports

A true (sad) story

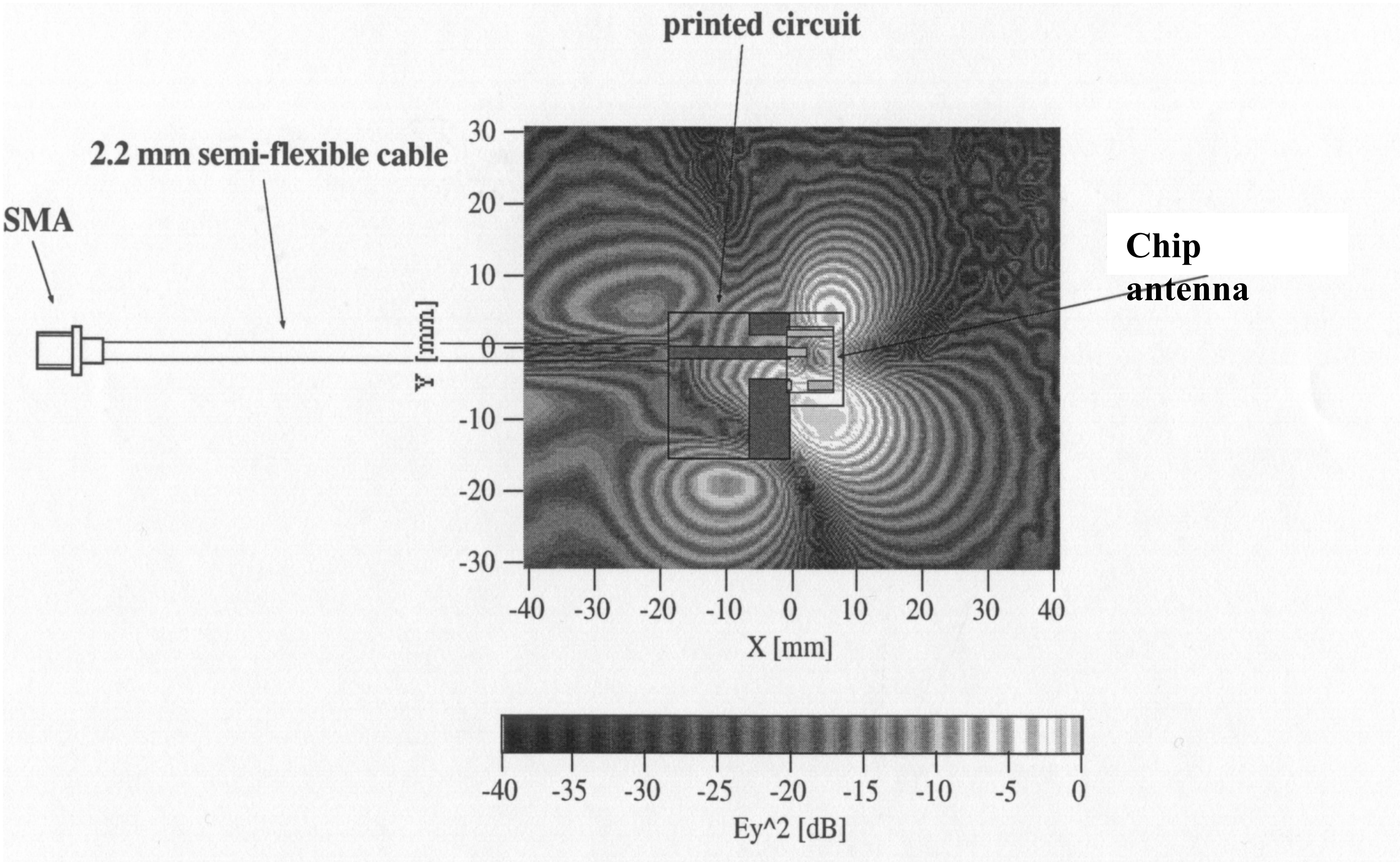
Electrically small antennas: What is the problem ?

- Commercial DECT antenna :
  - Ceramic chip, 6 x 9 x 1.8 mm
  - Gain of 2.2dBi at 1.89 GHz
  - Max Gain after Harrington : -3.3 dBi !!
  - Gain measured at LEMA : -8 ± 2 dBi
- The discrepancy is due to measurement errors

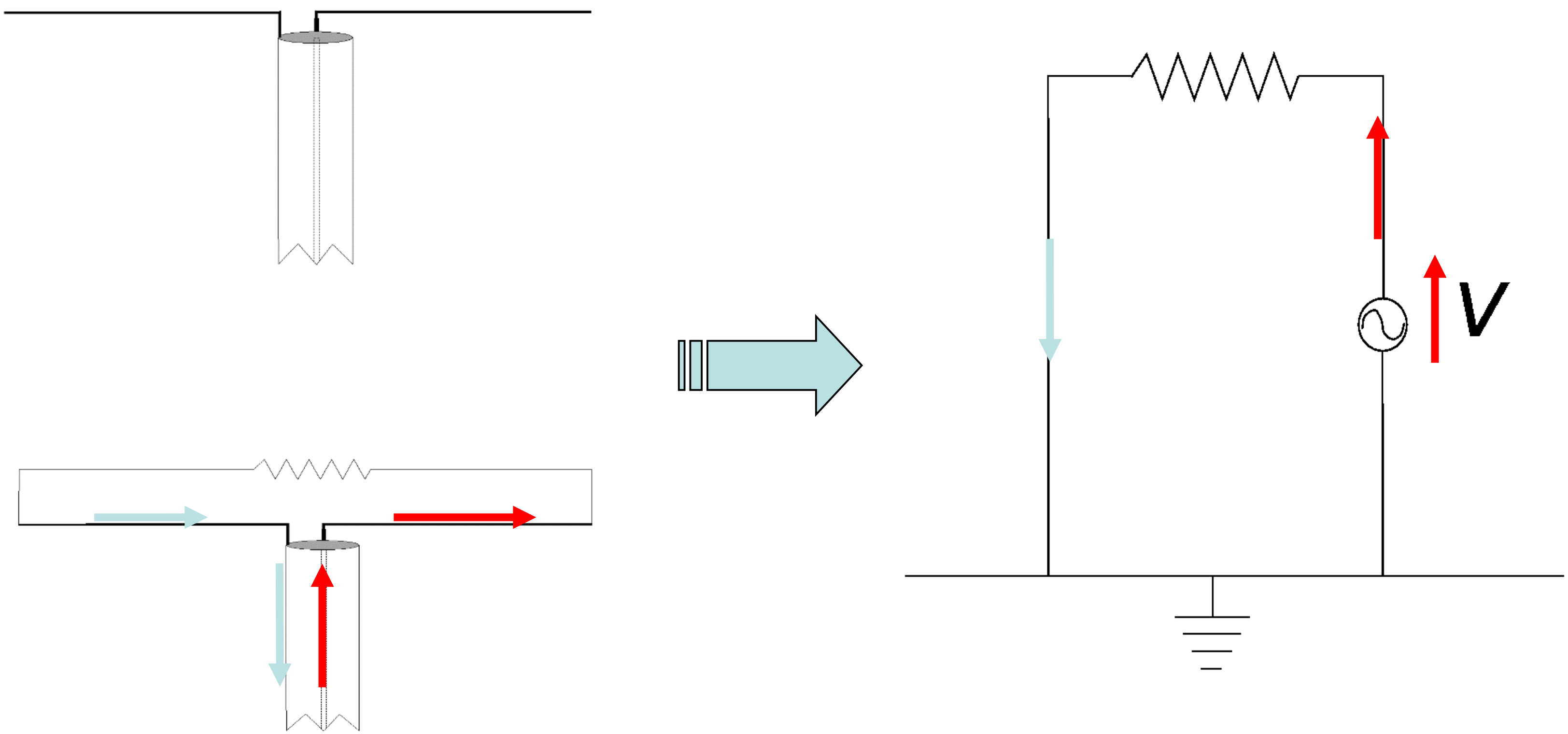
Effect of Spurious radiation in the case of the chip antenna



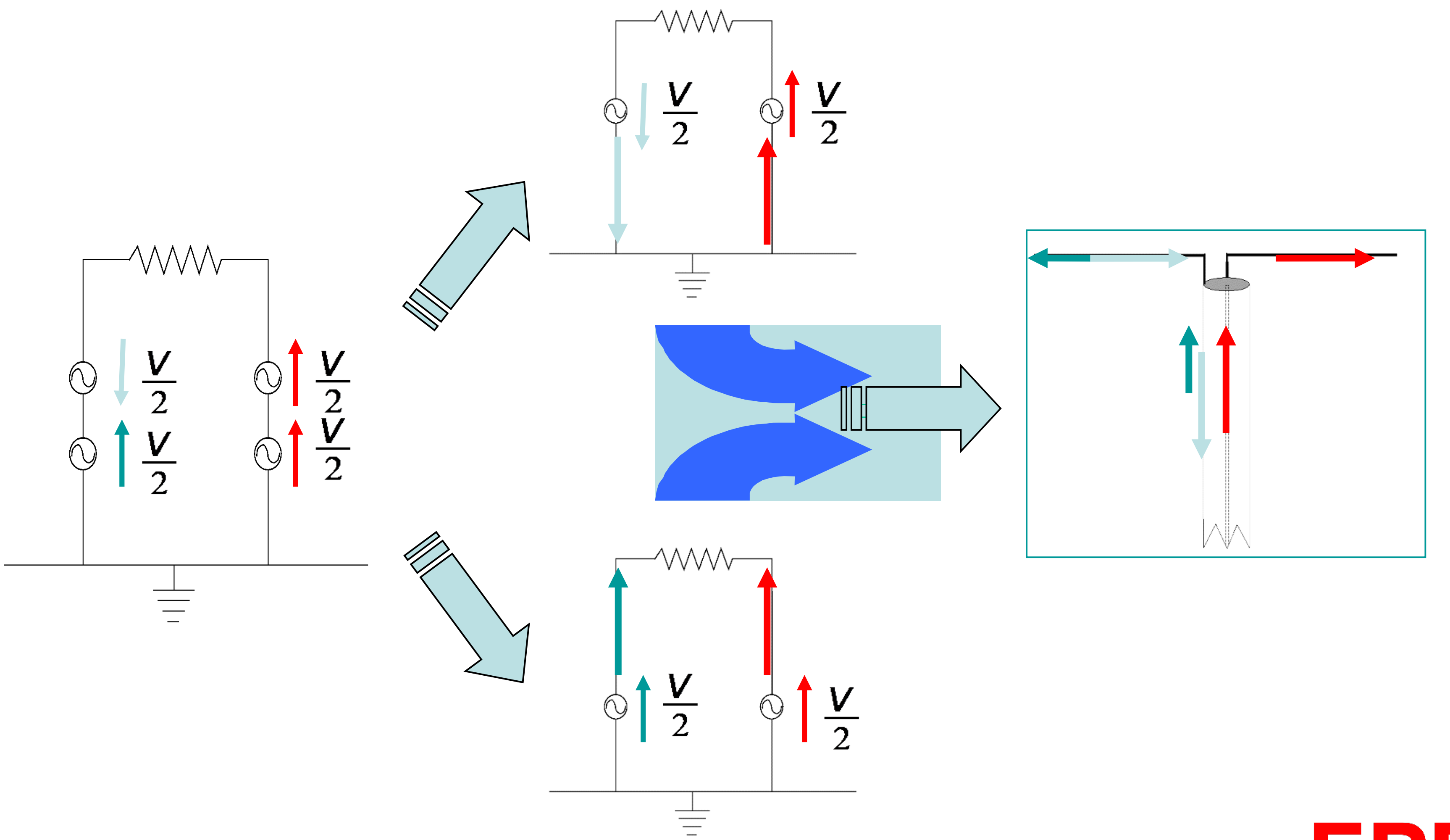
Effect of Spurious radiation in the case of the chip antenna



Spurious radiation from cables



Spurious radiation from cables



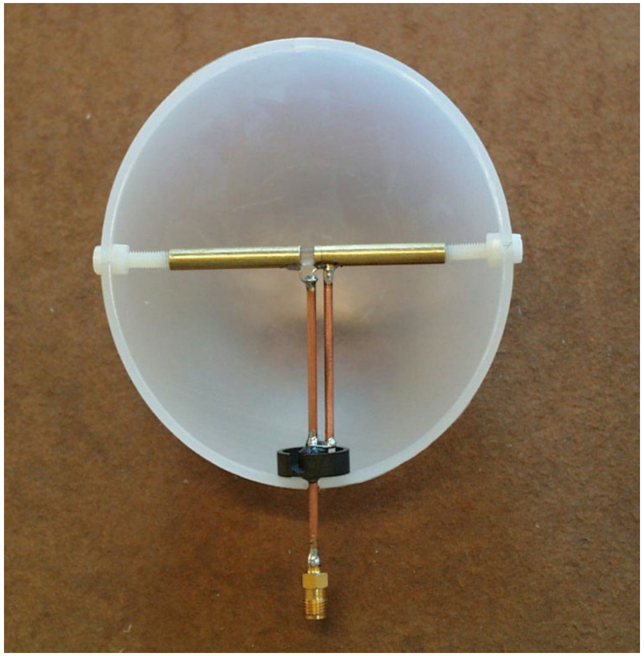
Effects on radiation characteristics

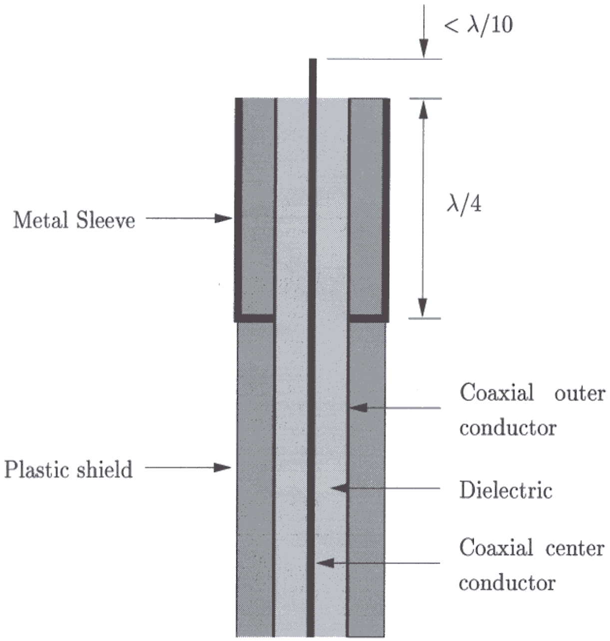
- Unwanted radiation in unwanted directions
- Increase of measured gain up to 10 dB
- Destruction of both polarization and radiation pattern

Measurement solutions

- Baluns
- Wheeler cap method
- System measurement methods
  - reverberation chamber
  - anechoic chamber

The spurious radiation can be attenuated using for instance ferrite cores, chockes or baluns.





Baluns

- Advantage :
  - good for both circuit and radiation measurements

- Disadvantages :
  - mostly narrow-band
  - cumbersome for the characterization of multi-band antennas

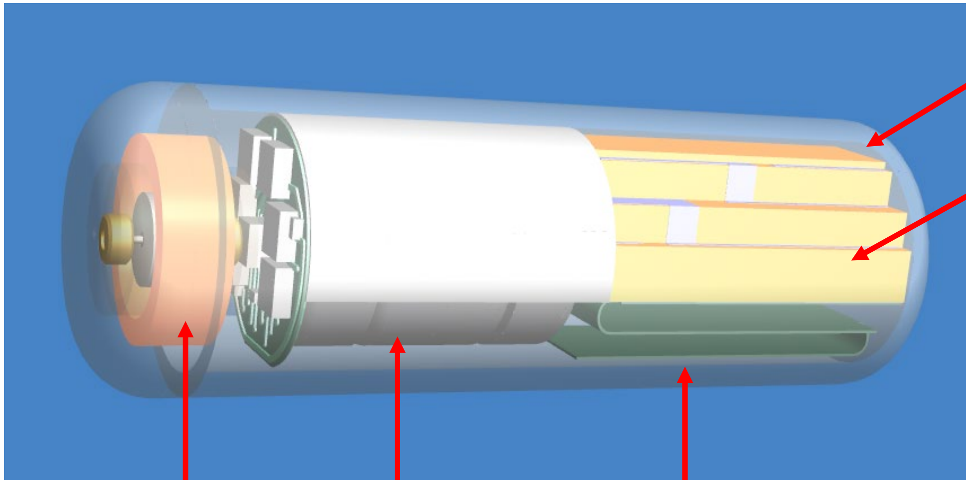
Measurement issues

- ESA are difficult to characterize as they do not have a proper port
- In certain cases, they cannot be measured as the connection to the cable modifies severely their characteristics
- In case of implantable antennas, the problem is worse due to the lossy environment. This is an old problem known from microwave hyperthermia.

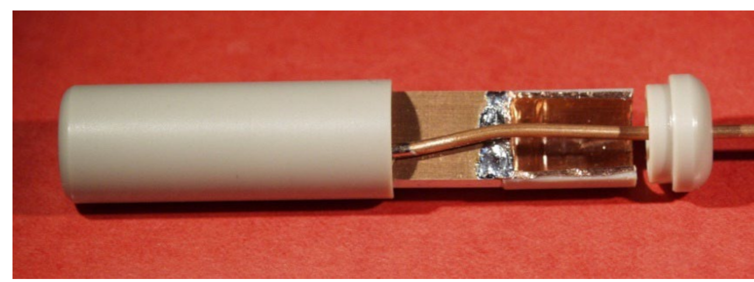
- But it is important to characterize the antenna before implanting the system !!!

- F. Merli and A.K.Skrivervik, Design and Measurement Considerations for Implantable Antennas for Telemetry Applications, EUCAP 2010, Barcelona

Illustration : a real case [1-2]



size 10x32mm



[1] F. Merli, L. Bolomey, J.-F. Zürcher, G. Corradini, E. Meurville, and A. K. Skrivervik, “Design, realization and measurements of a miniature antenna for implantable wireless communication systems,” *IEEE Trans. Antennas Propagat.*, submitted for publication.

[2] “Telemetry system for sensing applications in lossy media”, L. Bolomey; F. Merli; E. Meurville; J-F. Zürcher and A.K. Skrivervik, Patent number: 00335/10

Antenna conception

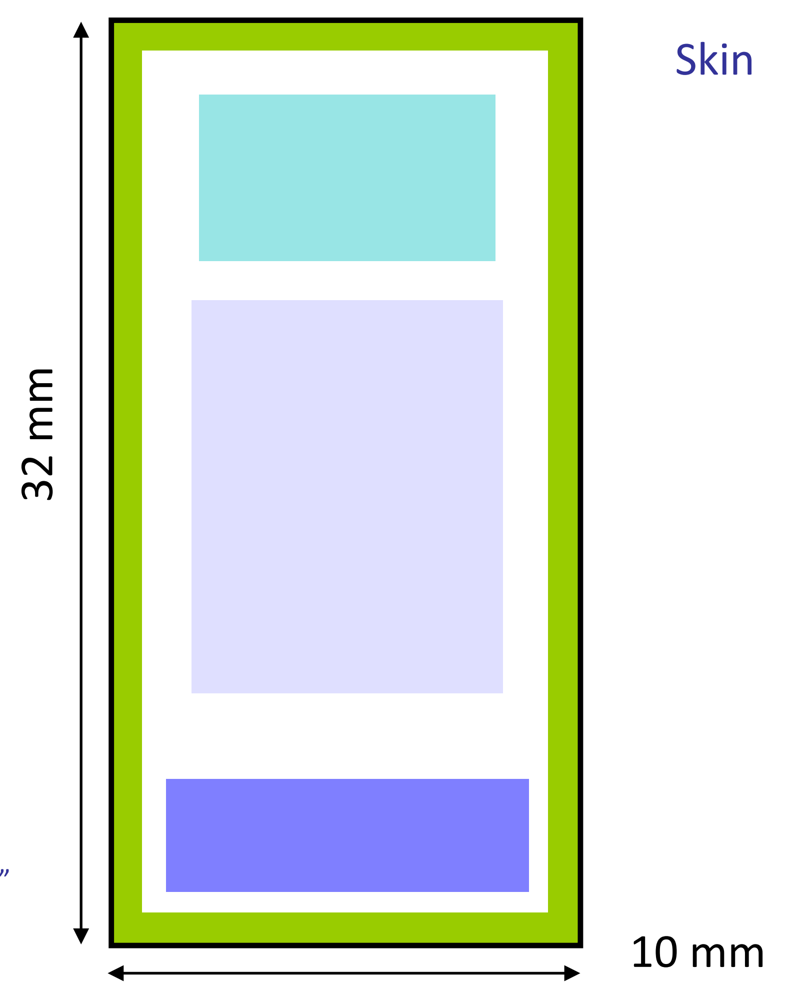
- Biocompatible insulation
- Circuitry
- Battery
- Sensor

a) Improve EM performance

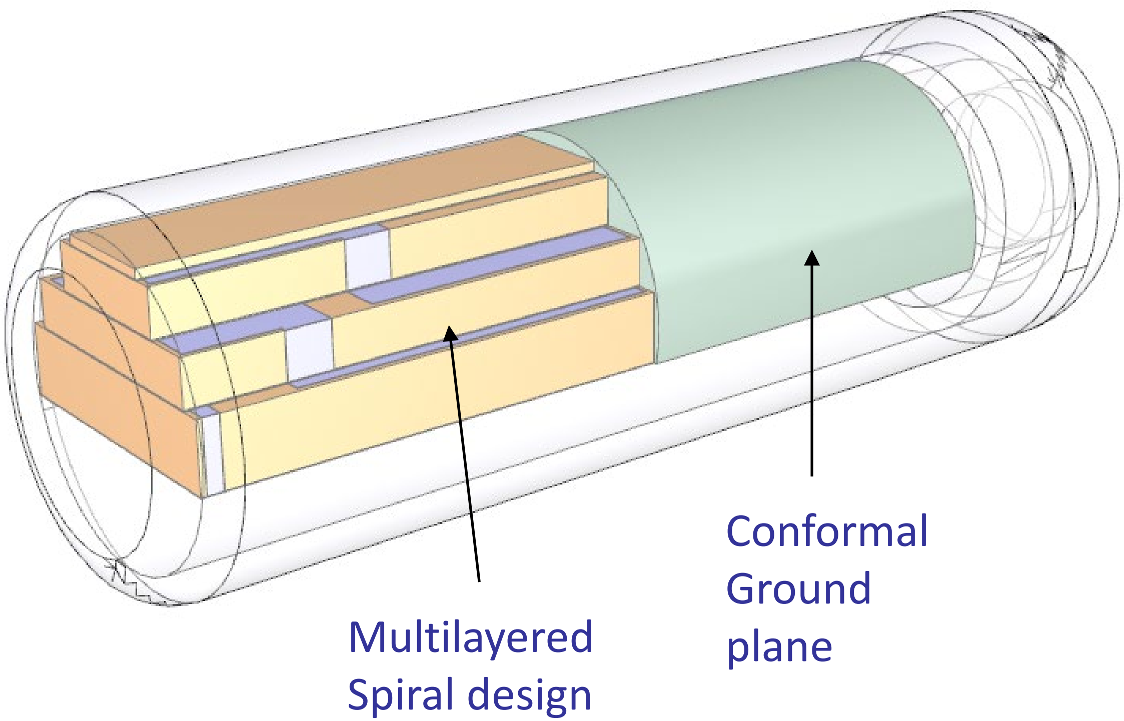
b) Enhance radiation out of the body

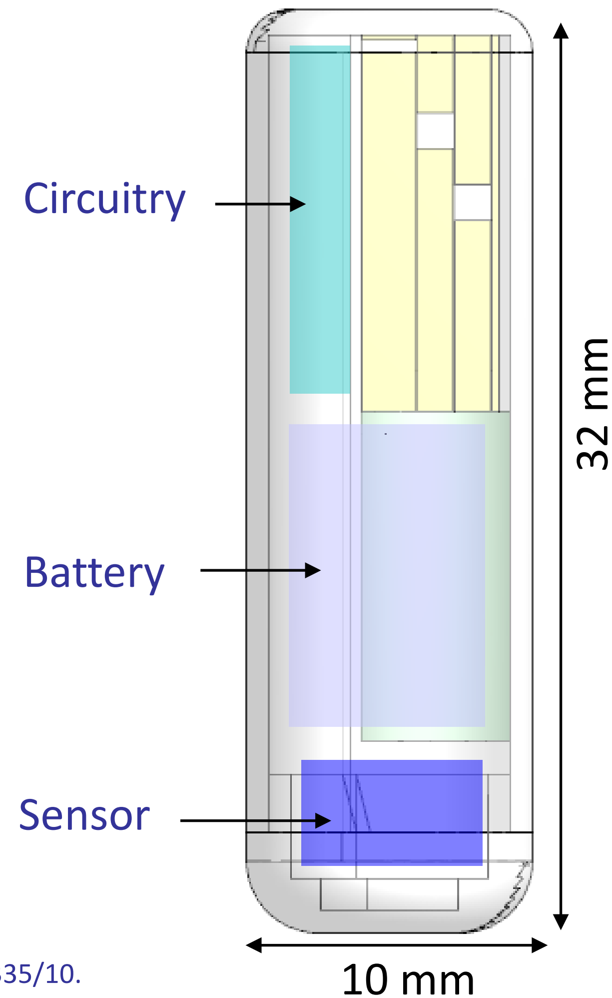
A. Barraud, "Molecular Selective Interface for an Implantable Glucose Sensor Based on the Viscosity Variation of a Sensitive Fluid Containing Dextran and Concanavalin A," Ph.D. Thesis, EPFL, Lausanne, 2008.

- F. Merli et al., "Implanted Antenna for Biomedical Applications," *AP-S 2008, San Diego*.



Antenna Structure



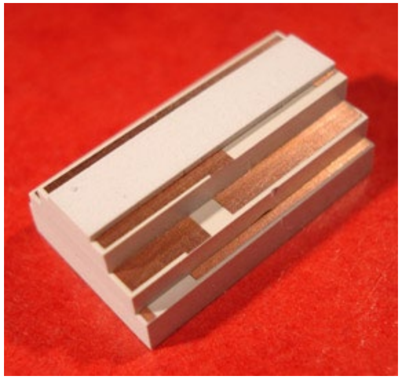


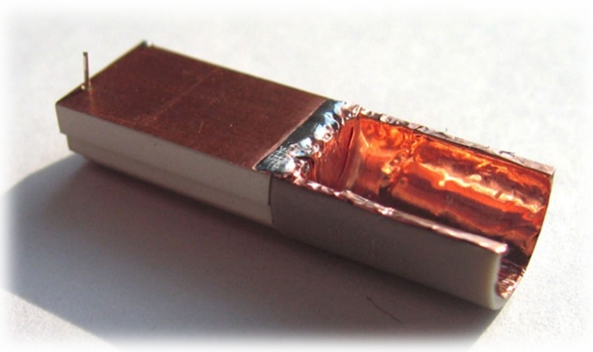
- F. Merli et al., "Design, realization and measurements of a miniature antenna for implantable wireless communication systems," *IEEE Trans. on AP*., submitted for publication.
- L. Bolomey et al., "Telemetry system for sensing applications in lossy media", Patent application: 00335/10.

Antenna Realization

Substrate: Roget TMM ($\varepsilon_r = 9.2$)

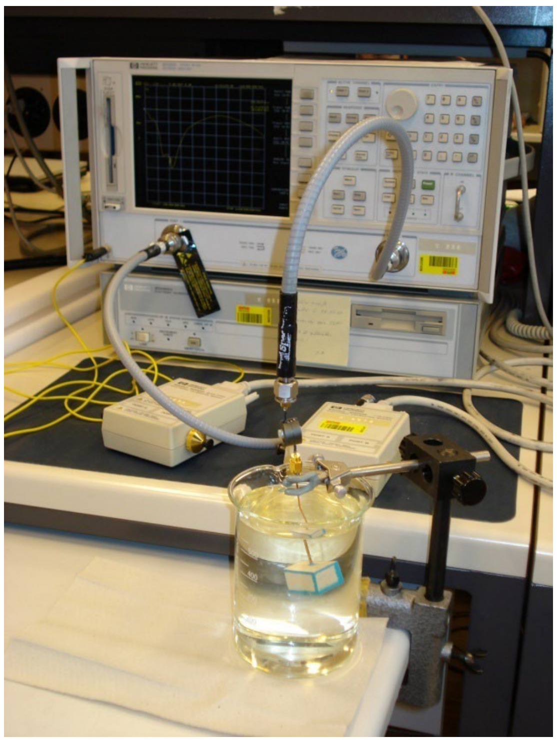
Insulation: PEEK ($\varepsilon_r = 3.2$)

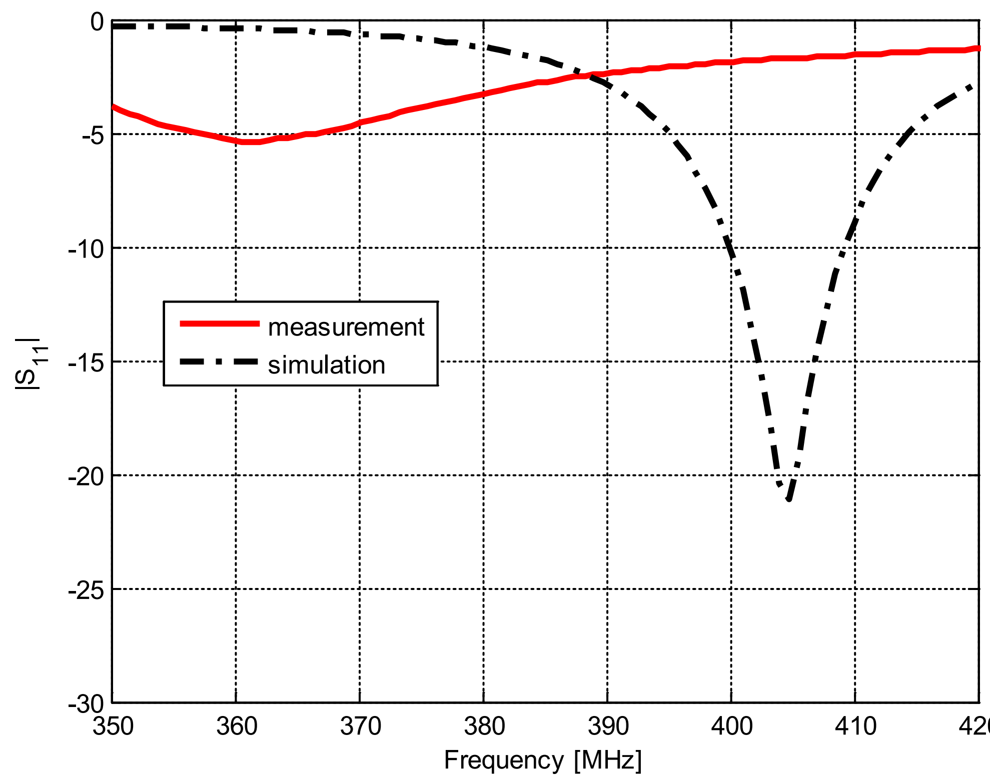




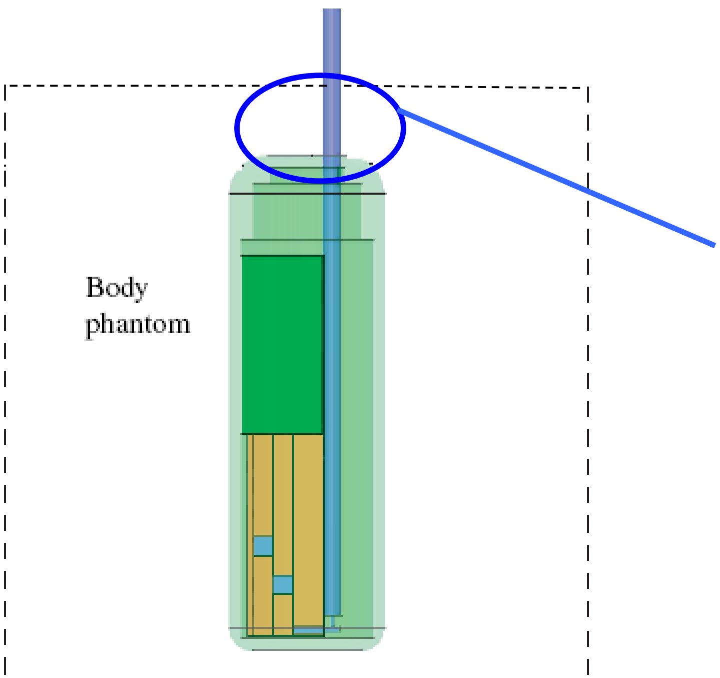
LEMA Miniature Antenna: First simulations and measurements

Electromagnetic performances of the antenna alone have been checked with a feeding coaxial cable (present only for testing purposes).





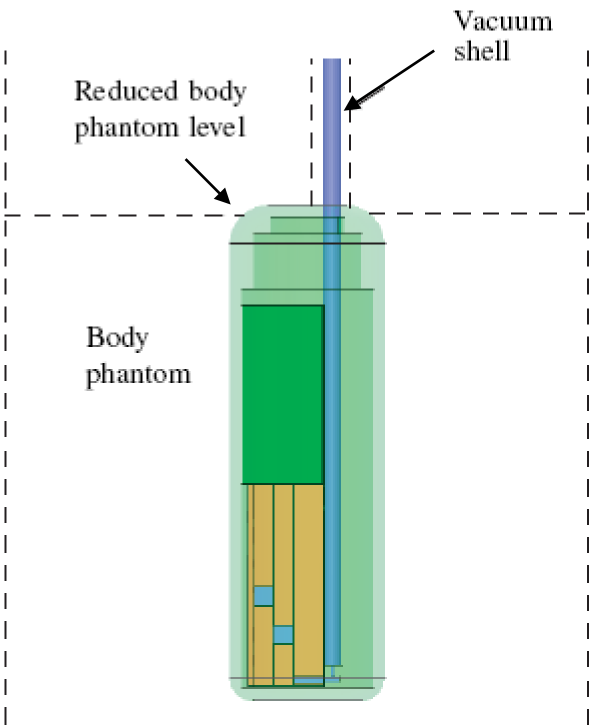
Main difference between simulation and measurement?

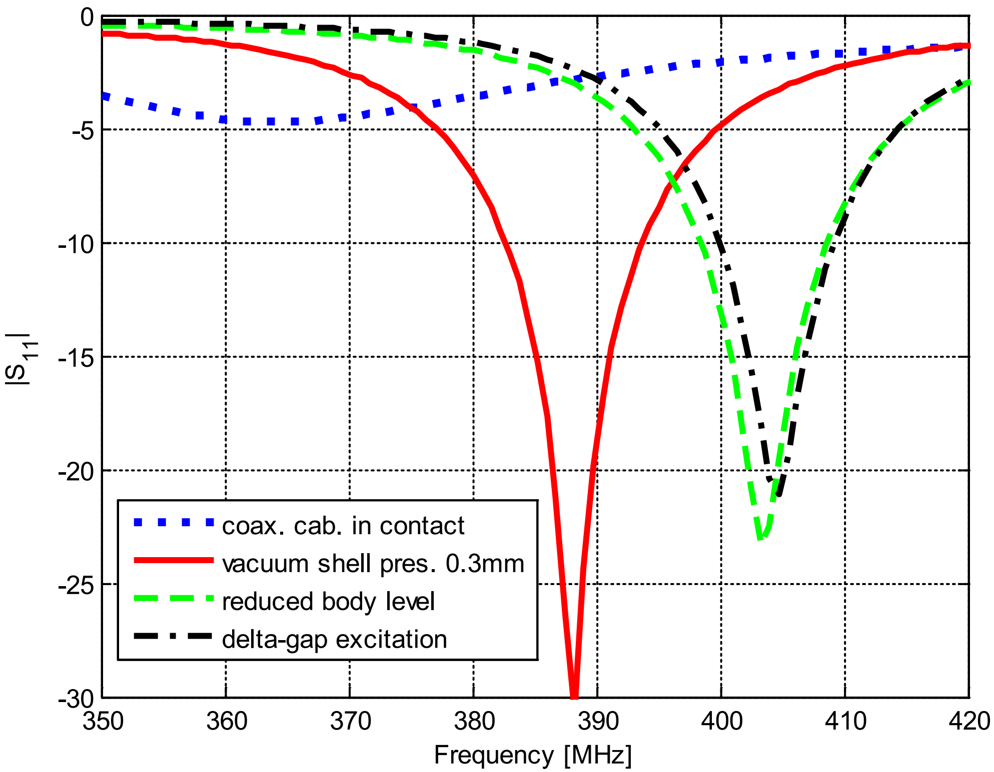


As is well known, the feeding coaxial cable affects the performances of electrically small antennas.

In the measurement setup, the coaxial cable is in direct contact with the biological liquid!!

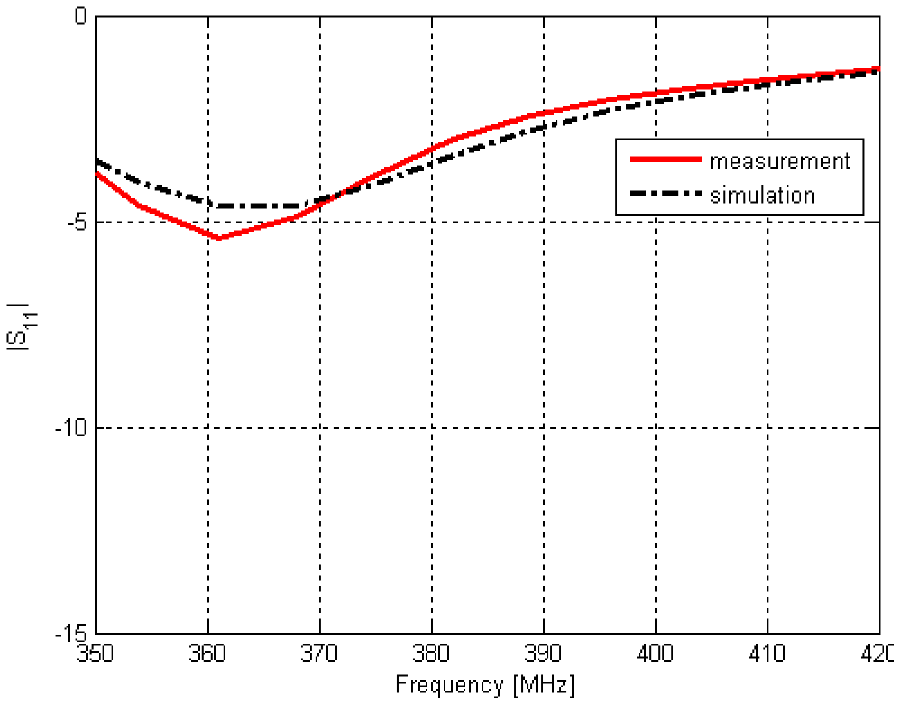
LEMA Miniature Antenna: how to mitigate the cable + body effect?

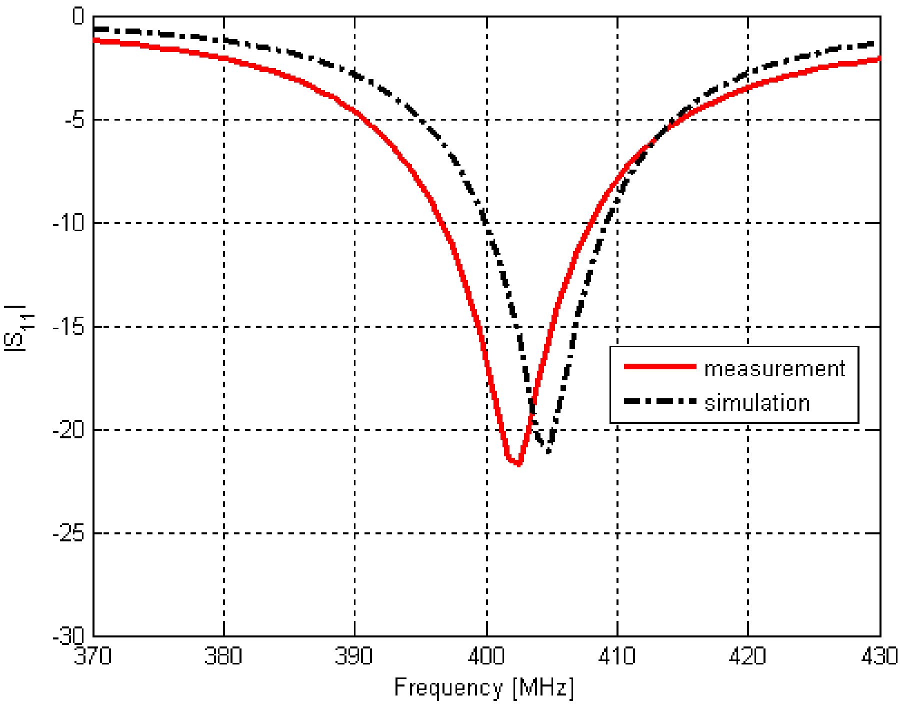




simulations

LEMA Miniature Antenna: how to mitigate the cable + body effect?





Coaxial cable in direct contact

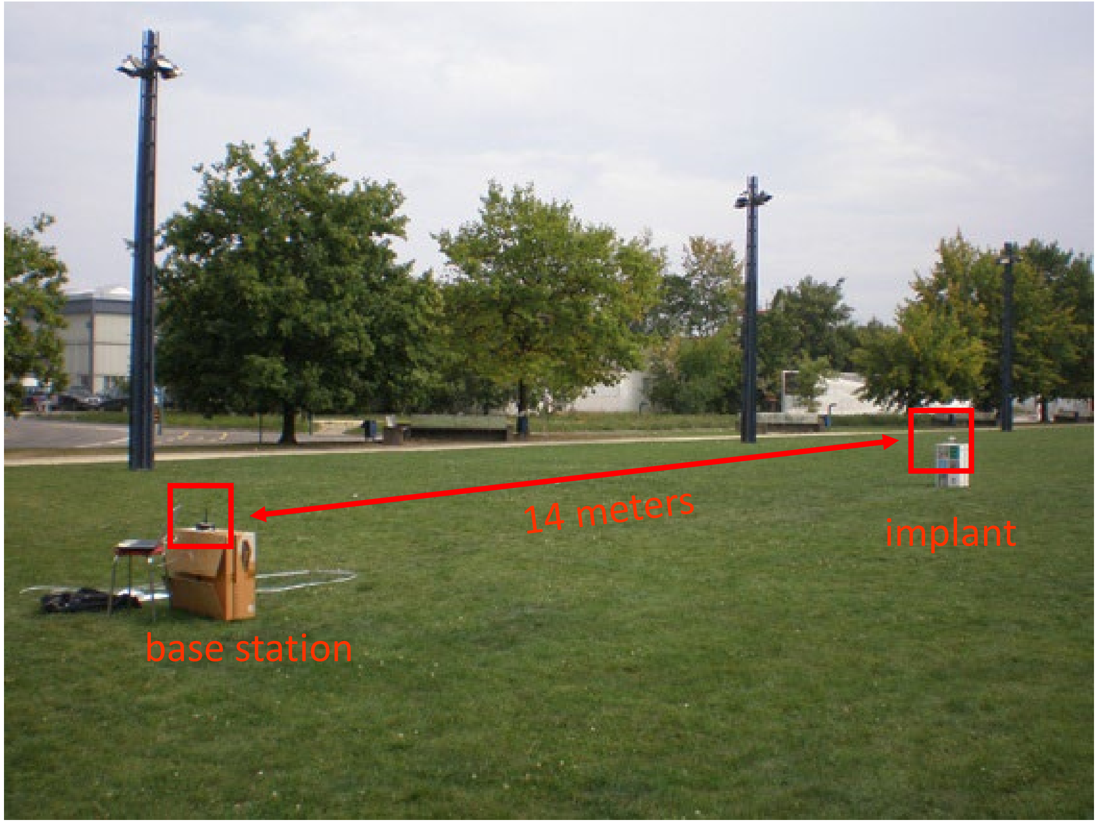
Reduced body phantom level and ideal excitation

System Measurements *In-vitro*

Zarlink BS is always considered
System controlled via a laptop (*Labview*)

Outdoor MedRadio Tests:
- TX power -3 dBm

channel	max range [m]
0	7
4	14
9	14



Implantation (*in collaboration with* the Stem Cell Dynamics Laboratory, LDCS):

Two devices have been implanted at different locations,

- subcutaneous (5 mm)
- in muscle tissue (30 mm)

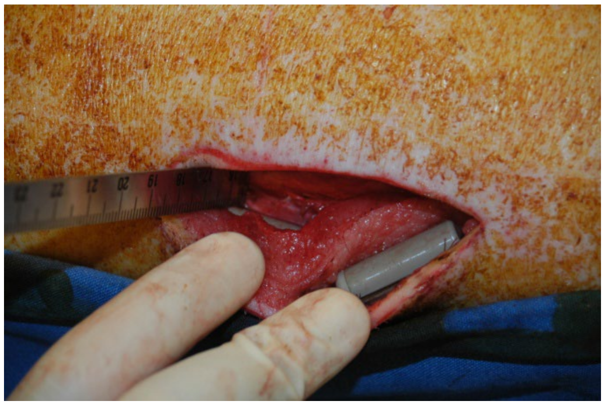
Target:

Continuous monitoring of subcutaneous temperature of a porcine animal (= pig)

Characteristics:

- Measurement during the implantation procedure
- Temperature check every 5 min. Complete working cycle (*wake-up, measurement, transmission…)* for 15 days

System *In-vivo*



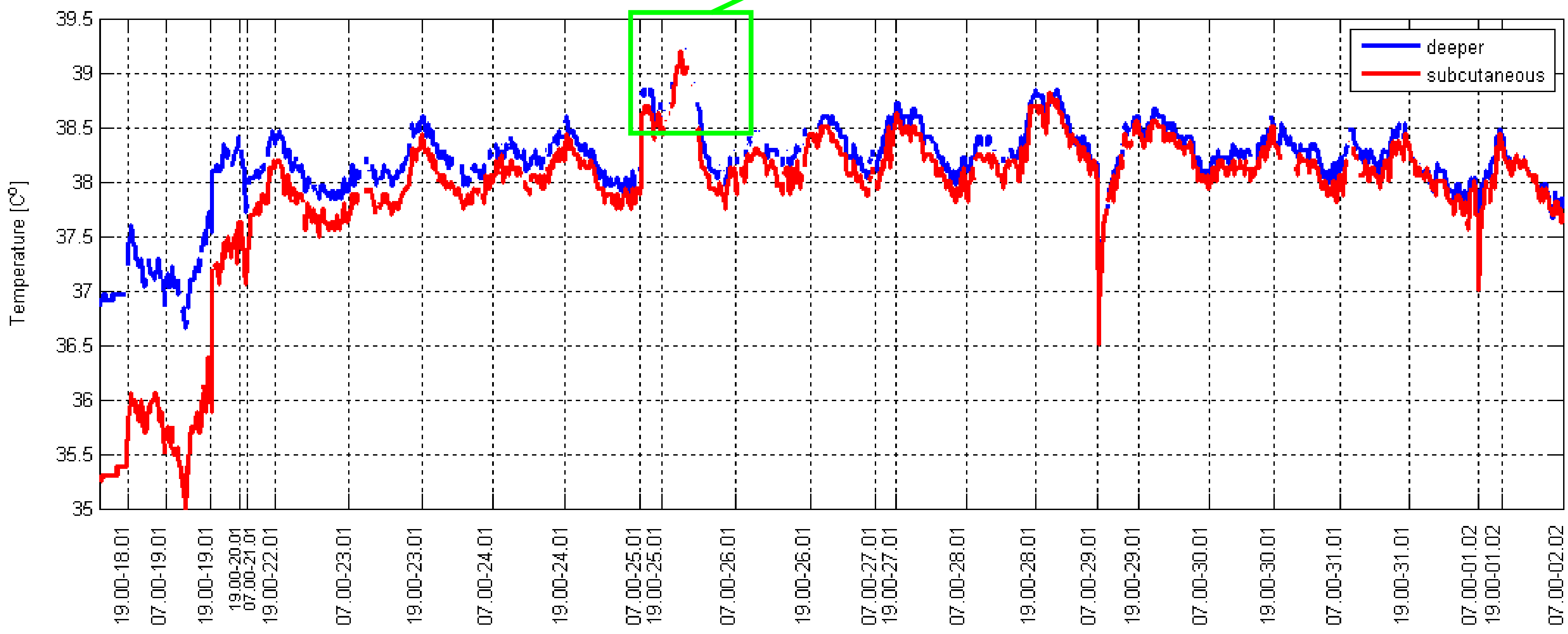
Implantation in accordance to all ethical considerations and the regulatory issues related to animal experiments.

In vitro sensor for room temperature comparison

System Measurements *In-vivo*

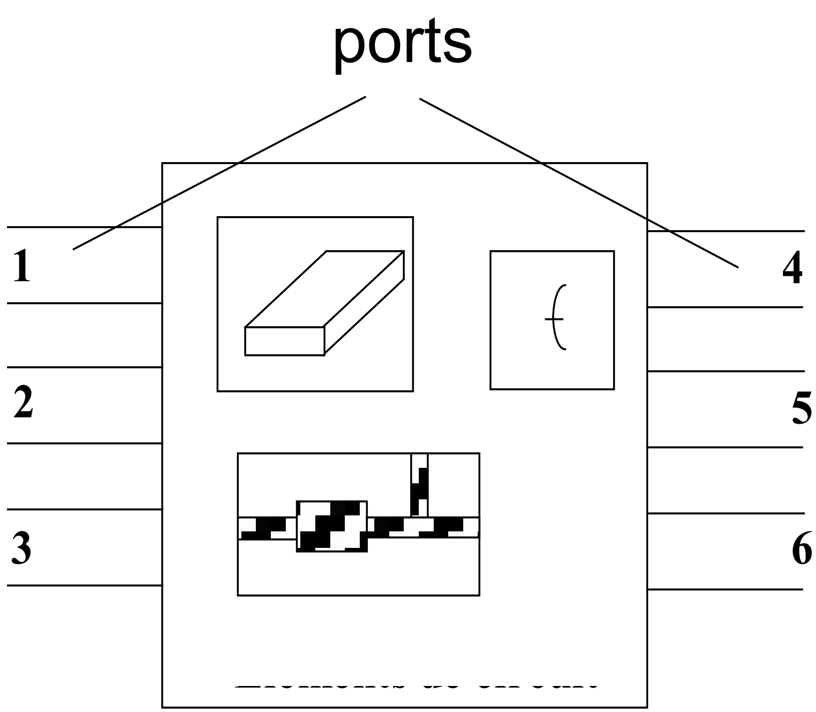
Highest relative error

(*biological explanation*)



End of interlude

Microwave component

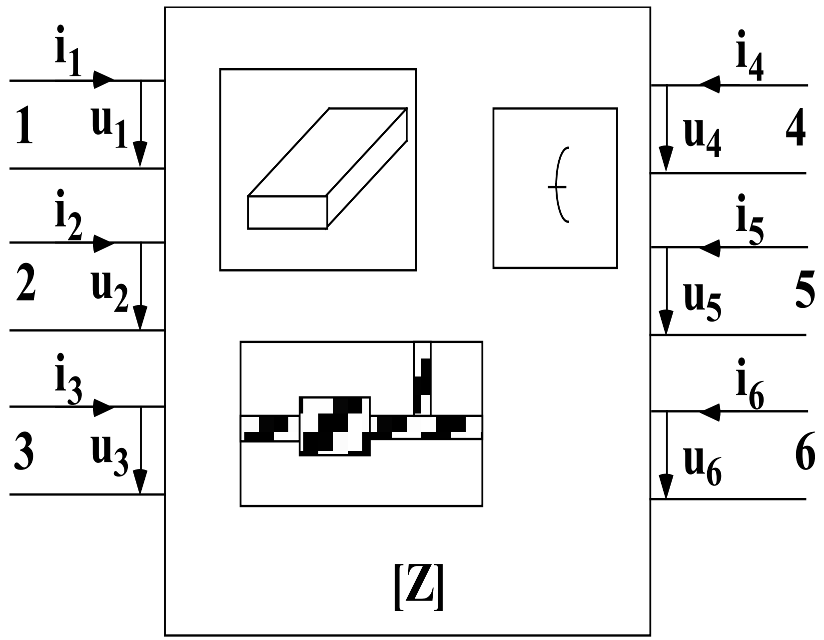


Circuit elements

Available cricuit models

- Kirchhoff
  - Currents and voltages are ill defined
  - Well known from low frequency circuit analysis
  - Not adapted to system approach
- Scattering matrix
  - Based on signal and power
  - Well suited to microwaves
  - Not known from low frequency circuit analysis
  - Adapted to system analysis

Kirchhoff's model



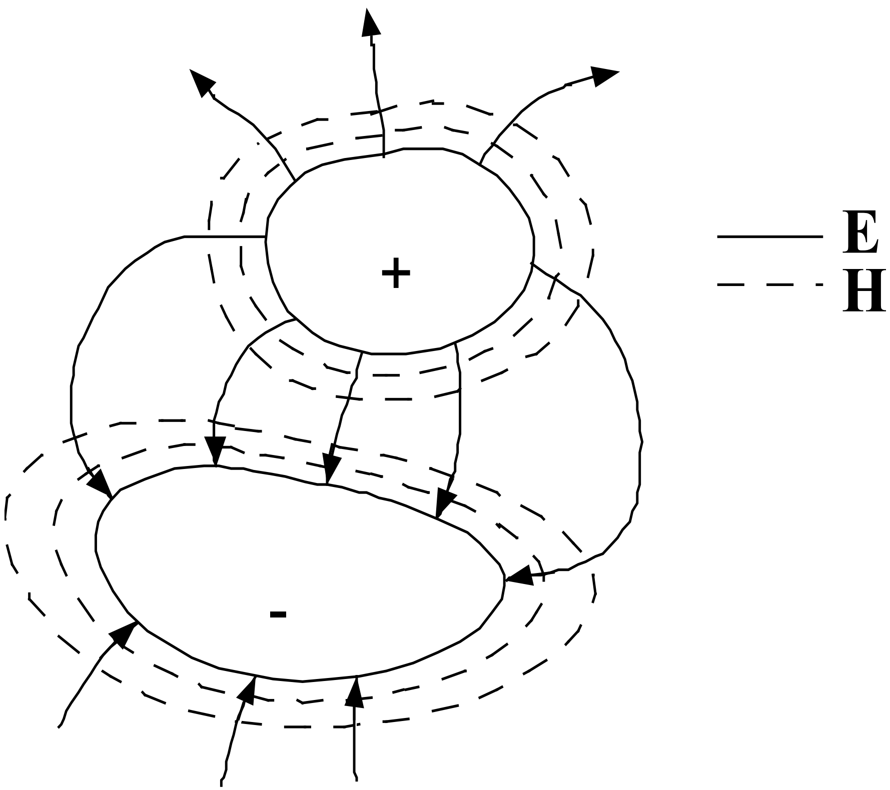
Current, voltages and impedances

- Easy to use
- Current and voltage difficult to measure
- Current and voltage not always uniquely defined

Example : TEM line

$$\nabla_t \times \mathbf{e_t} = 0 \qquad -\gamma\hat{\mathbf{z}} \times \mathbf{e_t} = -j\omega\mu\mathbf{h_t}$$

$$\nabla_t \times \mathbf{h_t} = 0 \qquad -\gamma\hat{\mathbf{z}} \times \mathbf{h_t} = j\omega\varepsilon\mathbf{e_t} \qquad \mathbf{e_t} = -\nabla_t V \quad \textit{with} \quad \nabla_t^2 V = 0$$



$$V = \int_{+}^{-} \mathbf{E} \cdot \mathbf{dl}$$

$$I = \oint_{C+} \mathbf{H} \cdot \mathbf{dl}$$

Characteristic impedance

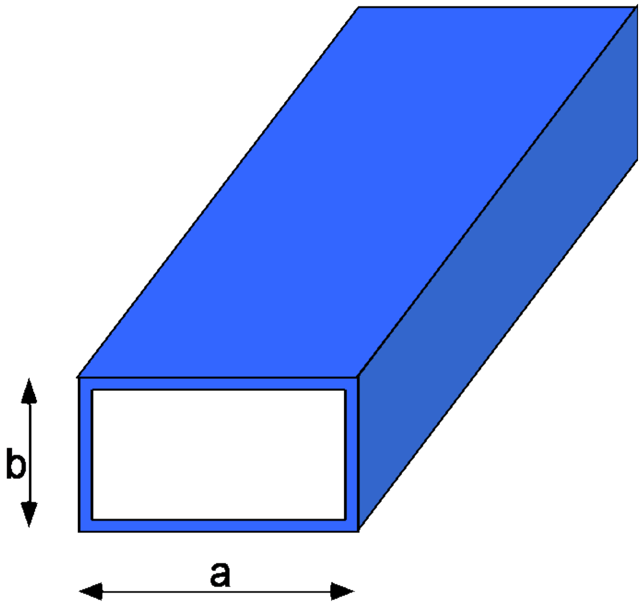
$$Z_c = \frac{V}{I} = \sqrt{\frac{L}{C}}$$

Example : TEM line

- Univoque definition of voltage and current
- Fields identical to static fields
- Zero cutoff frequency
- Needs at least two separate conductors

Non-TEM line : the rectangular waveguide

$$\mathbf{E} = -\nabla V - j\omega\mu\mathbf{A}$$



Example : mode TE10 of a rectangular waveguide

$$E_y(x,y,z) = E_0 \frac{j\omega\mu a}{\pi} \sin\frac{\pi x}{a} e^{-j\beta z} = E_0 e_y(x,y) e^{-j\beta z}$$

$$H_x(x,y,z) = E_0 \frac{j\beta a}{\pi} \sin\frac{\pi x}{a} e^{-j\beta z} = E_0 h_y(x,y) e^{-j\beta z}$$

Thus

$$V = E_0 \frac{-j\omega\mu a}{\pi} \sin\frac{\pi x}{a} e^{-j\beta z} \int_y dy$$

Non TEM voltage and current definition

- Only defined for one mode
- The voltage is proportional to the transverse electric field
- The current is proportional to the transverse magnetic field
- The characteristic impedance is equal to U/I. We choose the characteristic impedance equal to the wave impedance
- The power flux is given by the product of the voltage and the current

Arbitrary line

$$\mathbf{E_t}(x,y,z) = \mathbf{e_t}(x,y)\left(E_o^+ e^{-j\beta z} + E_o^- e^{j\beta z}\right)$$

$$= \frac{\mathbf{e_t}(x,y)}{C_1}\left(V^+ e^{-j\beta z} + V^- e^{j\beta z}\right)$$

$$\mathbf{H_t}(x,y,z) = \mathbf{h_t}(x,y)\left(E_o^+ e^{-j\beta z} - E_o^- e^{j\beta z}\right)$$

$$= \frac{\mathbf{h_t}(x,y)}{C_2}\left(I^+ e^{-j\beta z} - I^- e^{j\beta z}\right)$$

$$V(z) = V^+ e^{-j\beta z} + V^- e^{j\beta z}$$

$$I(z) = I^+ e^{-j\beta z} - I^- e^{j\beta z}$$

$$Z_c = \frac{V^+}{I^+} = \frac{V^-}{I^-} = \frac{C_1 E_o^+}{C_2 E_o^+} = \frac{C_1}{C_2}$$

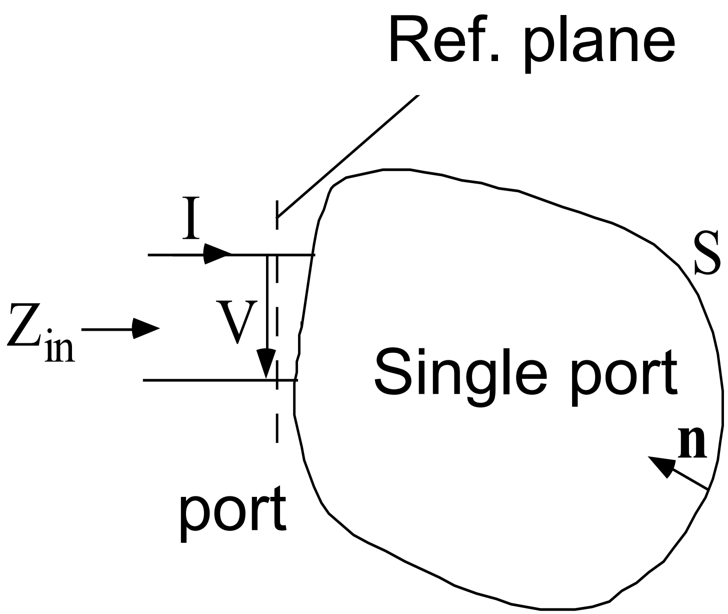
$$\frac{C_1}{C_2} = Z_{\mathrm{mod}}$$

Different impedances

- Impedance of the medium : $$Z_o = \sqrt{\frac{\mu}{\varepsilon}}$$
- Wave impedance : $$Z_{\text{mod}} = \frac{|\mathbf{E_t}|}{|\mathbf{H_t}|}$$
- Line characteristic impedance : $$Z_c = \frac{V^+}{I^+} = \frac{V^-}{I^-} = \sqrt{\frac{L}{C}}$$
- Impedance matrix of a circuit

Impedance matrix

Single port element



$$Z_{in} = \frac{V}{I}$$

Power delivered to the component (Poynting)

$$P = \oint_S \mathbf{E} \times \mathbf{H}^* \cdot \mathbf{ds} = P_r + j\omega\left(W_m - W_e\right)$$

Single port element

Voltage and current :

$$\mathbf{E_t}(x,y,z) = V(z)\frac{\mathbf{e_t}(x,y)}{C_1}e^{-j\beta z}$$

$$\mathbf{H_t}(x,y,z) = I(z)\frac{\mathbf{h_t}(x,y)}{C_2}e^{-j\beta z}$$

with $\frac{1}{C_1 C_2}\int_s \mathbf{e_t} \times \mathbf{h_t} \cdot \mathbf{ds} = 1$ thus $P = \frac{1}{C_1 C_2}\int_s VI^* \mathbf{e_t} \times \mathbf{h_t} \cdot \mathbf{ds} = VI^*$

and

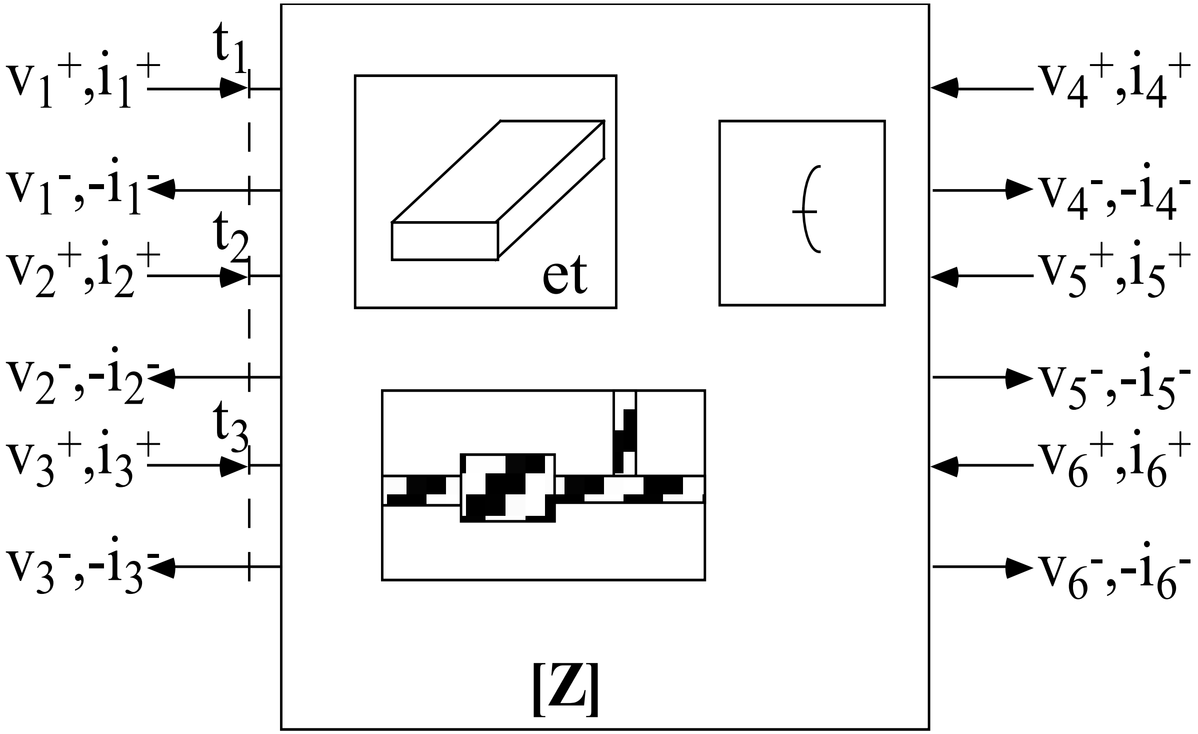
$$Z_{in} = R + jX = \frac{V}{I} = \frac{VI^*}{|I|^2} = \frac{P}{|I|^2}$$

$$= \frac{\left(P_r + j\omega\left(W_m - W_e\right)\right)}{|I|^2}$$

Single port element

- R is proportional to to the real power dissipated in the system (Losses)
- X is proportional to the mean reactive stored energy in the system

Impedance and admittance matrix



$$V_n = V_n^+ + V_n^-$$

$$I_n = I_n^+ - I_n^-$$

At each port t_i

Impedance and admittance matrix

$$[V]=[Z][I]$$
$$[I]=[Y][V]$$

$$Z_{ij}=\left.\frac{V_i}{I_j}\right|_{I_k=0 \quad pour \quad k\neq j}$$

$$Y_{ij}=\left.\frac{I_i}{V_j}\right|_{V_k=0 \quad pour \quad k\neq j}$$

- Impedance matrix is obtained in open circuit conditions
- Admittance matrix is obtained in short circuit conditions

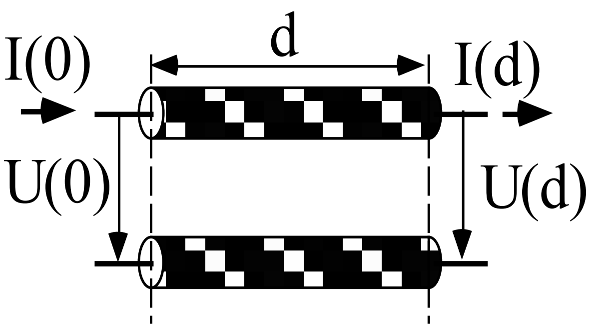
Properties of the impedance and admittance matrix

- Reciprocity : $Y_{ij} = Y_{ji}$

  $Z_{ij} = Z_{ji}$

- Lossless circuit : $\mathrm{Re}\{Z_{mn}\} = 0$

Example : transmission line

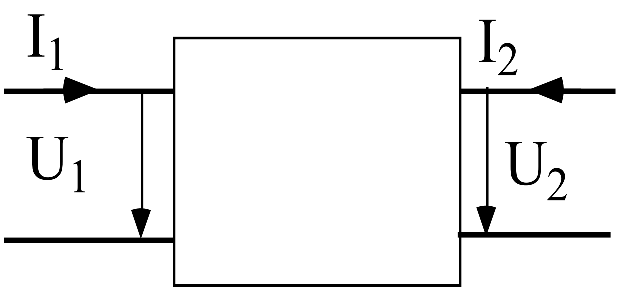


$$U_1 = U(0), I_1 = I(0),$$

$$U_2 = U(d), I2 = -I(d)$$

$$U(z) = U_+ \mathrm{e}^{-\gamma z} + U_- \mathrm{e}^{+\gamma z}$$

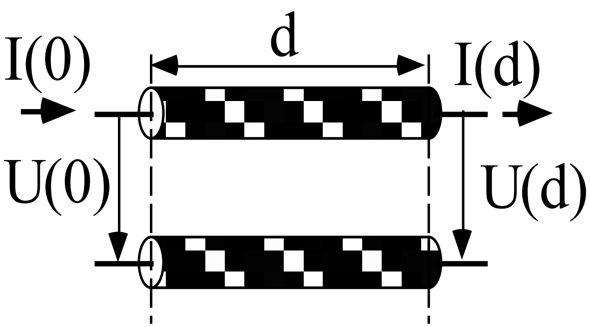
$$I(z) = I_+ \mathrm{e}^{-\gamma z} - I_- \mathrm{e}^{+\gamma z}$$

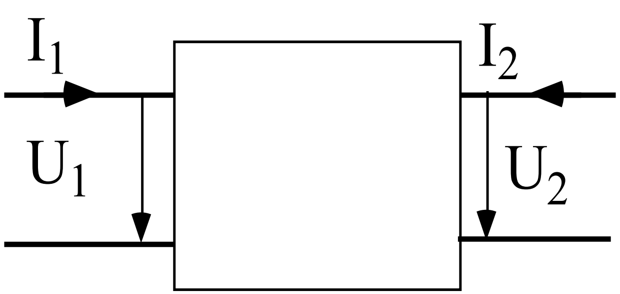


$$U(0) = U_+ + U_- \quad U(d) = U_+ \mathrm{e}^{-\gamma d} + U_- \mathrm{e}^{+\gamma d}$$

$$I(0) = I_+ - I_- \quad I(d) = I_+ \mathrm{e}^{-\gamma d} - I_- \mathrm{e}^{+\gamma d}$$

Example : transmission line





$$\begin{bmatrix} U_1 \\ U_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

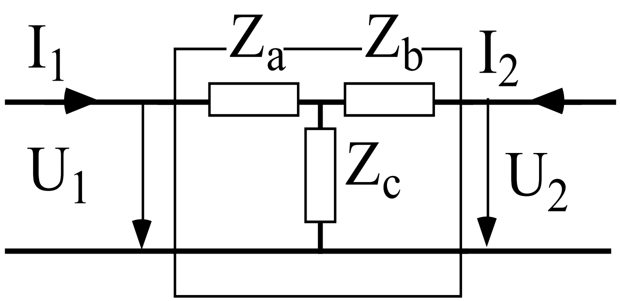
$$= Z_c \begin{bmatrix} \coth(\gamma d) & \dfrac{1}{\sinh(\gamma d)} \\ \dfrac{1}{\sinh(\gamma d)} & \coth(\gamma d) \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \end{bmatrix}$$

$$= Y_c \begin{bmatrix} \coth(\gamma d) & \dfrac{-1}{\sinh(\gamma d)} \\ \dfrac{-1}{\sinh(\gamma d)} & \coth(\gamma d) \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \end{bmatrix}$$

Equivalent T circuit of a reciprocal two-port

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{12} & Z_{22} \end{bmatrix}$$

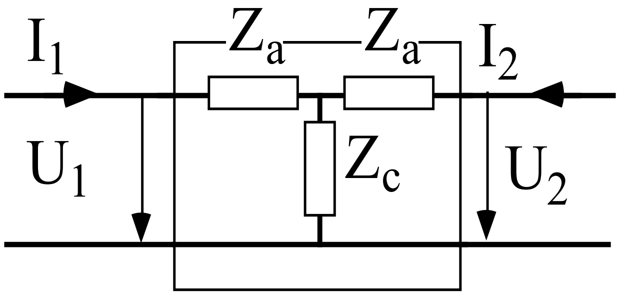


$$Z_a = Z_{11} - Z_{12}$$

$$Z_b = Z_{22} - Z_{12}$$

$$Z_c = Z_{12}$$

Equivalent T circuit of a transmisson line



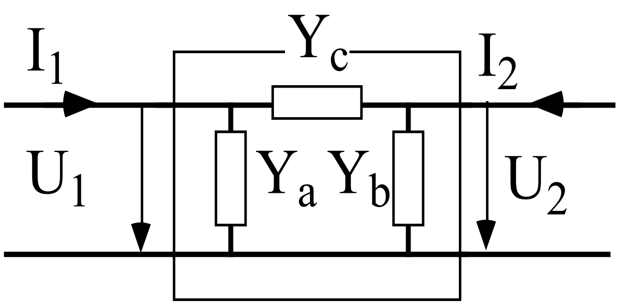
$$Z_a = Z_{caract}\left(\coth(\gamma d) - \frac{1}{\sinh(\gamma d)}\right)$$

$$= Z_{caract}\left(\frac{ch(\gamma d)-1}{\sinh(\gamma d)}\right) = Z_{caract}\tanh\left(\frac{\gamma d}{2}\right)$$

$$Z_c = \frac{Z_{caract}}{\sinh(\gamma d)}$$

Equivalent Π circuit of a reciprocal two-port

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{12} & Y_{22} \end{bmatrix}$$

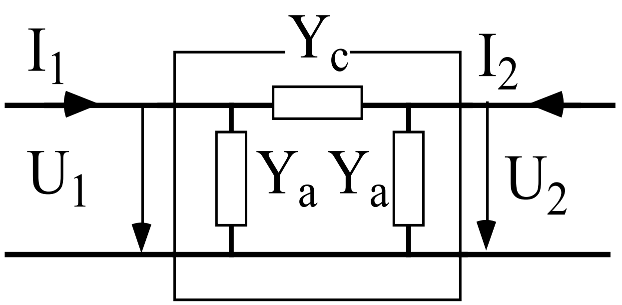


$$Y_a = Y_{11} + Y_{12}$$

$$Y_b = Y_{22} + Y_{12}$$

$$Y_c = -Y_{12}$$

Equivalent Π circuit of a transmisson line



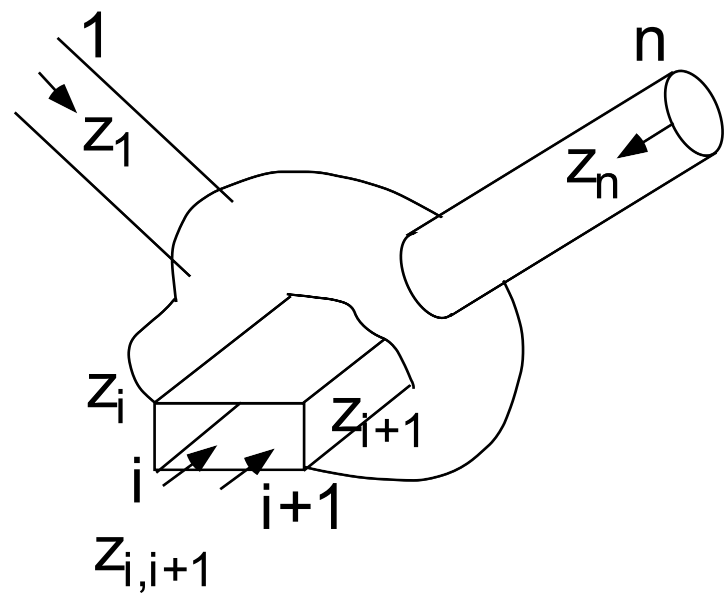
$$Y_a = Y_{caract}\left(\coth(\gamma d) - \frac{1}{\sinh(\gamma d)}\right)$$

$$= Y_{caract}\left(\frac{ch(\gamma d) - 1}{\sinh(\gamma d)}\right) = Y_{caract}\tanh\left(\frac{\gamma d}{2}\right)$$

$$Y_c = \frac{Y_{caract}}{\sinh(\gamma z)}$$

Impedance matrix in a multi-mode case

- Each mode corresponds to a separate port
- The other rules concerning reference planes continue to apply



The dimension of the impedance matrix will be given by

$$N = \sum_{i=1,nline} k_i$$

Where nline is the number of access lines and k_i the number of modes propagated by line i

Scattering parameters

Normalized wave amplitudes : definition

Normalized wave amplitudes

$$a_i = \frac{v_i + Z_{ci} i_i}{2\sqrt{Z_{ci}}} \quad , \quad b_i = \frac{v_i - Z_{ci} i_i}{2\sqrt{Z_{ci}}} \qquad [W^{1/2}]$$

Inverse relation

$$v_i = \sqrt{Z_{ci}}\left(a_i + b_i\right) \quad , \quad i_i = \frac{\left(a_i - b_i\right)}{\sqrt{Z_{ci}}}$$

Normalized wave amplitudes : properties

Transmission lines

$$v_i = v_i^+ e^{-j\beta z} + v_i^- e^{+j\beta z}$$

$$i_i = i_i^+ e^{-j\beta z} + i_i^- e^{+j\beta z}$$

$$\longrightarrow$$

$$a_i = \frac{v_i^+}{\sqrt{Z_{ci}}} e^{-j\beta z}$$

$$b_i = \frac{v_i^-}{\sqrt{Z_{ci}}} e^{+j\beta z}$$

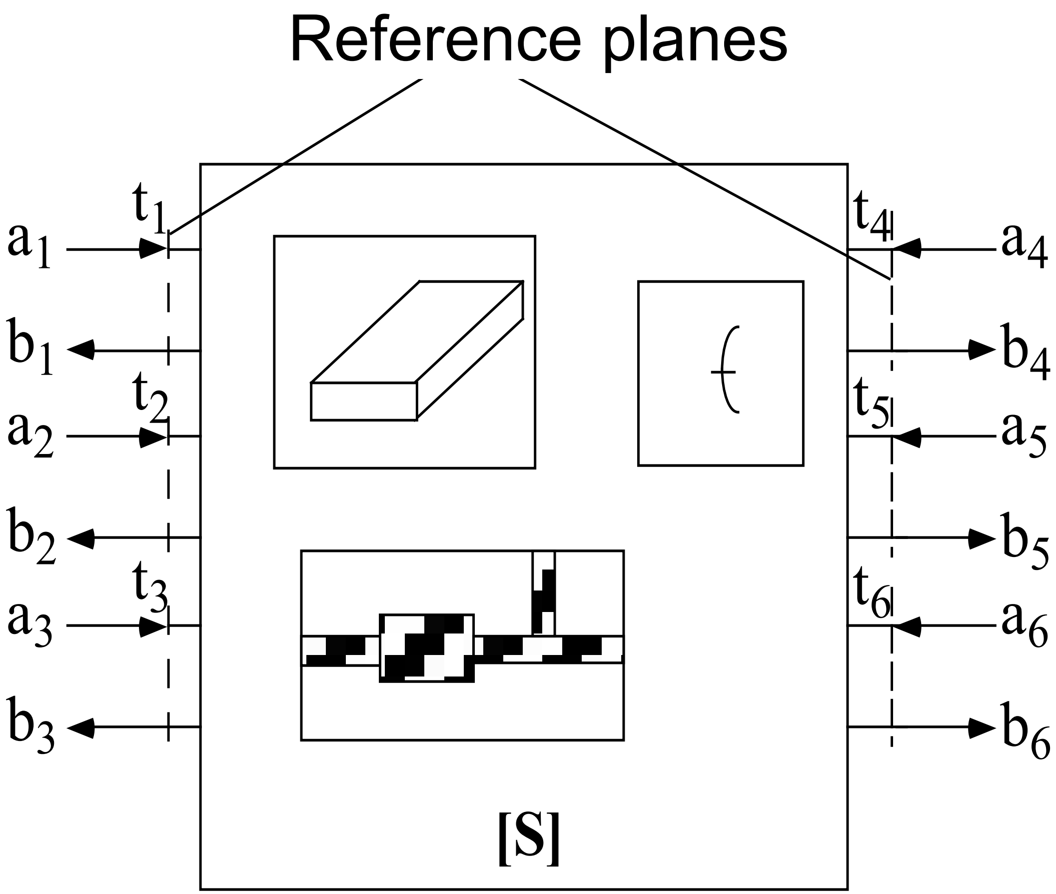
a_i : progressive wave
b_i : retrograde wave

Power at one component port

$$P_i = \mathrm{Re}\left[v_i i_i^*\right] = \mathrm{Re}\left[\left(a_i + b_i\right)\left(a_i^* - b_i^*\right)\right] = \left|a_i\right|^2 - \left|b_i\right|^2$$

$/a_i/^2$: power entering port i
$/b_i/^2$: power leaving port i

Signal model



$$[b] = [S][a]$$

$$s_{ij} = \left.\frac{b_i}{a_j}\right|_{a_k = 0\ ,\ k \neq j}$$

Scattering matrix : definition

Normalized wave amplitude

$$a_i = \frac{v_i + Z_{ci} i_i}{2\sqrt{Z_{ci}}} \ , \ b_i = \frac{v_i - Z_{ci} i_i}{2\sqrt{Z_{ci}}}$$

Inverse relation

$$v_i = \sqrt{Z_{ci}}\left(a_i + b_i\right) \ , \ i_i = \frac{\left(a_i - b_i\right)}{\sqrt{Z_{ci}}}$$

Scattering matrix

$$[b] = [S][a] \qquad s_{ij} = \left.\frac{b_i}{a_j}\right|_{a_k = 0 \ , \ k \neq j}$$

Signal model

- Advantages :
  - Signals are easy to measure
  - Well suited to a system approach (transfer functions)
- Drawbacks :
  - Not known at low frequencies

Scattering matrix : properties

- Sij : transfer function from port j to port i
- Depends on the component and its properties
- To change the connecting lines implies to change the scattering matrix
- The scattering matrix is obtained terminating the ports by matched loads

Reciprocity

Passive, linear, isotropic circuit

$\Downarrow$

The circuit is reciprocal

$\Downarrow$

$$z_{ij} = z_{ji}$$

$\Downarrow$

$$s_{ij} = s_{ji}$$

Lossless circuit

$$\sum |a_i|^2 = \sum |b_i|^2 \longrightarrow [\tilde{a}][a] - [\tilde{b}][b] = 0$$

With $[\tilde{a}] = [a^*]^t$

Moreover $[b] = [S][a]$, $[\tilde{b}] = [\tilde{a}][\tilde{S}]$ $\longrightarrow$

$$[\tilde{a}][a] - [\tilde{a}][\tilde{S}][S][a] = 0$$

$$[\tilde{a}]\{[1] - [\tilde{S}][S]\}[a] = 0$$

$$[\tilde{S}][S] = [1]$$

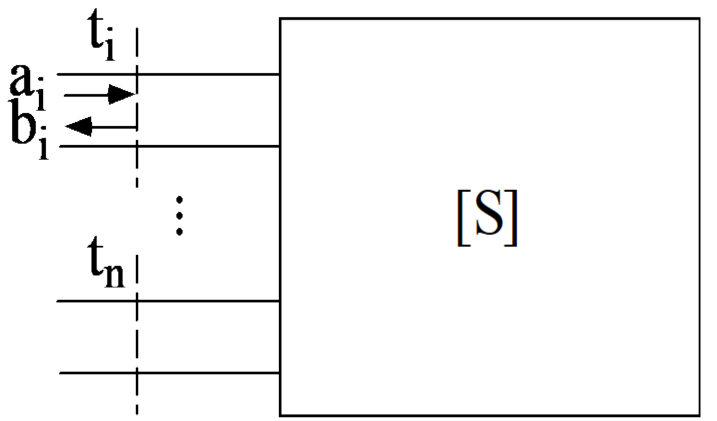
$\longrightarrow$

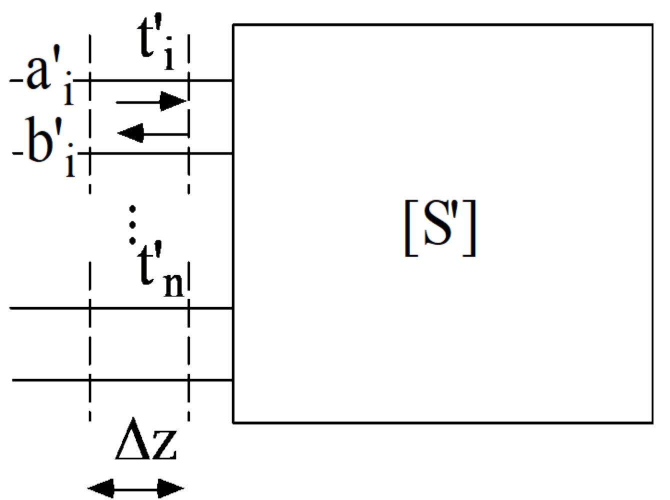
$$\sum_{i=1}^{N} s_{ij}^* s_{ik} = \delta_{jk} \qquad \delta_{jk} = \begin{cases} 1 & si \quad j = k \\ 0 & si \quad j \neq k \end{cases}$$

Matched circuit

$S_{ii} = 0$

Change of reference plane





Change of reference plane

$$a'_i = a_i e^{-j\varphi_i}$$

$$b'_i = b_i e^{j\varphi_i}$$

$$\varphi_i = -\beta_i \Delta z_i$$

thus $$s'_{ii} = s_{ii} e^{j2\varphi_i}$$

And in general

$$[a] = \left[diag\ e^{j\varphi}\right][a']$$

$$[a'] = \left[diag\ e^{-j\varphi}\right][a]$$

$$[b] = \left[diag\ e^{-j\varphi}\right][b']$$

$$[b'] = \left[diag\ e^{j\varphi}\right][b]$$

$$\left[diag\ e^{j\varphi}\right] = \begin{bmatrix} e^{j\varphi_1} & 0 & \dots & 0 \\ 0 & e^{j\varphi_2} & & : \\ : & & & \\ 0 & \dots & & e^{j\varphi n} \end{bmatrix}$$

Change of reference plane

$$[b'] = \left[diag\, \mathrm{e}^{j\varphi} \right] [S] \left[diag\, \mathrm{e}^{j\varphi} \right] [a']$$

$$[S'] = \left[diag\, \mathrm{e}^{j\varphi} \right] [S] \left[diag\, \mathrm{e}^{j\varphi} \right]$$

$$s'_{ij} = s_{ij}\, \mathrm{e}^{j\left(\varphi_i + \varphi_j\right)}$$

Passive circuits : questions

1. Linearity
2. Reciprocity
3. Losslessness
4. Matched
5. Symmetry

[S]->[Z]

$[\underline{b}] = [\underline{S}]\,[\underline{a}]$ $\quad \underline{a}i = (\underline{U}i + Zci\,\underline{I}i)/2\sqrt{Zci}$ $\quad \underline{U}i = \sqrt{Zci}\,(\underline{a}i + \underline{b}i)$

$[\underline{U}] = [\underline{Z}]\,[\underline{I}]$ $\quad \underline{b}i = (\underline{U}i - Zci\,\underline{I}i)/2\sqrt{Zci}$ $\quad \underline{I}i = (\underline{a}i - \underline{b}i)/\sqrt{Zci}$

We define two auxiliary matrices

$[G] = [\mathrm{diag}(Zci)]$ $\qquad [F] = [\mathrm{diag}(1/2\sqrt{Zci})]$

And we can write

$[\underline{a}] = [F]\,\{[\underline{U}] + [G]\,[\underline{I}]\}$ $\qquad ([\underline{a}] + [\underline{b}]) = 2\,[F]\,[\underline{U}]$

$[\underline{b}] = [F]\,\{[\underline{U}] - [G]\,[\underline{I}]\}$ $\qquad ([\underline{a}] - [\underline{b}]) = 2\,[F]\,[G]\,[\underline{I}]$

[S]->[Z]

$$[\underline{a}] = [F] \{[\underline{U}] + [G] [\underline{I}] \} \rightarrow [\underline{a}] = [F] \{[\underline{Z}] + [G] \} [\underline{I}]$$

$$\rightarrow [\underline{I}] = \{[\underline{Z}] + [G] \}^{-1} [F]^{-1} [\underline{a}]$$

$$[\underline{b}] = [F] \{[\underline{U}] - [G] [\underline{I}] \} \rightarrow [\underline{b}] = [F] \{[\underline{Z}] - [G] \} [\underline{I}]$$

$$\rightarrow [\underline{b}] = [F] \{[\underline{Z}] - [G] \} \{[\underline{Z}] + [G] \}^{-1} [F]^{-1} [\underline{a}] = [\underline{S}] [\underline{a}]$$

Thus: $[\underline{S}] = [F] \{[\underline{Z}] - [G] \} \{[\underline{Z}] + [G] \}^{-1} [F]^{-1}$

In one dimension, we have $\underline{\rho} = \underline{S}11 = (\underline{Z}t - Zc)/(\underline{Z}t + Zc)$

$([\underline{a}] + [\underline{b}]) = 2\,[F]\,[\underline{U}]$ **->** $[\underline{U}] = (2\,[F])^{-1}([\underline{a}] + [\underline{b}]) = (2\,[F])^{-1}([1] + [\underline{S}])\,[\underline{a}]$

$([\underline{a}] - [\underline{b}]) = 2\,[F]\,[G]\,[\underline{I}] = ([1] - [\underline{S}])\,[\underline{a}]$ **->** $[\underline{a}] = ([1] - [\underline{S}])^{-1}\,2\,[F]\,[G]\,[\underline{I}]$

we group the similar terms:

$$[\underline{U}] = (2\,[F])^{-1}([1] + [\underline{S}])\,([1] - [\underline{S}])^{-1}\,2\,[F]\,[G]\,[\underline{I}] = [\underline{Z}]\,[\underline{I}]$$

Et on en tire: $[\underline{Z}] = ([F])^{-1}([1] + [\underline{S}])\,([1] - [\underline{S}])^{-1}\,[F]\,[G]$

For a single port component we obtain :

$$\underline{Z}t = Zc(1 + \underline{r})/(1 - \underline{r})$$